

COT 3100H Section 201 Exam #2 3/5/2024

Last Name: _____, **First Name:** _____

1) (10 pts) Prove or disprove: if p and q are prime numbers, then at least one of the following five numbers is also prime: $pq - 2$, $pq - 1$, pq , $pq + 1$, $pq + 2$.

2) (10 pts) What is the largest power of 12 that divides evenly into $1000!$? Please show your work for full credit and put a box around your final answer.

3) (15 pts) (a) Find the ordered pair of integers (x, y) that satisfies $248x + 111y = 3$, with the minimum possible **positive** value of x .

(b) In addition, determine $111^{-1} \bmod 248$. (Your answer must be a positive integer less than 248.)

(a) (_____ , _____)

(b) _____

4) (12 pts) (a) Plug in values to make a guess for a closed-form solution to the following summation, in terms of n : $\sum_{k=1}^n k(k!)$. (Copy the summation, put an equal sign and put your formula in terms of n on the right hand side.)

(b) Use mathematical induction to prove that is your guess is correct for all positive integers n .

5) (12 pts) Define the recurrence relation $t(n)$ for all non-negative integers as follows:

$$t(0) = 4, t(1) = 11, t(n) = 7t(n-1) - 10t(n-2), \text{ for all } n \geq 2.$$

Prove via strong induction on n with 2 base cases that for all non-negative integers n ,

$$t(n) = 3 \times 2^n + 5^n$$

6) (12 pts) Prove, via induction on n , that for all non-negative integers n , $7 \mid (3 + 4 \times 8^{2n})$.

7) (8 pts) 3 workers, each working at the same uniform rate, take 5 days to build 2 widgets. How many days would it take 2 of these workers, working at the same uniform rate, to build 7 of the same widgets? Please express your answer as a mixed fraction in lowest terms.

8) (10 pts) Jenna does 5 push ups on day 1. She vows to complete exactly 2 more push ups on each subsequent day than the day before. On which day number does she complete 107 push ups? How many total push ups has she done starting on day 1 through the day she did 107 push ups?

Day with 107 push ups: _____ , Total Number of push ups: _____

9) (10 pts) Let x be the smallest integer with precisely 35 divisors. It turns out that there exists a positive integer y such that $x = y^2$. With proof, determine the value of y . (Put a box around your answer.)

10) (1 pt) The famous Boston Massacre took place on March 5, 1770 (exactly 254 years ago). In which city did it occur?

Scratch Page – Please clearly label any work on this page you would like graded.