

Spring 2017 COT 3100 Exam #2 - Sheet 1 (3/23/2017)

Last Name: _____ , **First Name :** _____

Lab Section: 17 18 19 20 21 22 24

- 1) (15 pts) Using induction on n , prove for all non-negative integers n , that $10 \mid (9^{n+1} + 7^{2n})$.

2) (15 pts) Let r and n be positive integers with $r \leq n$. Using induction on n , prove the following for all positive integers n , where r is an arbitrary positive integer less than or equal to n :

$$\sum_{i=r}^n \binom{i}{i-r} = \binom{n+1}{n-r}$$

Note: You may use (without including the proof), the following identity proven in class:

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

where $0 < k < n$. Hint: You should leave r as it is throughout the entire proof as this formula is true for any r one might choose that is positive and less than or equal to n . The proof is relatively short, shorter than one might expect upon first viewing the problem.

Since the proof is quite short, please clearly use words to justify the algebra in each step.

Spring 2017 COT 3100 Exam #2 - Sheet 2 (3/23/2017)

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3) (15 pts) Using induction on n, prove for all non-negative integers n, that:

$$\begin{bmatrix} 5 & -6 \\ 1 & 0 \end{bmatrix}^n \begin{bmatrix} 16 \\ 6 \end{bmatrix} = \begin{bmatrix} 2^{n+2} + 4(3^{n+1}) \\ 2^{n+1} + 4(3^n) \end{bmatrix}$$

4) (5 pts) How many permutations are there of the letters HALLOWEDHALLS? (Just write down your answer, no explanation necessary.)

5) (10 pts) What is the coefficient of x^7 in the expansion of $(2x - 3)^{10}$? Leave your answer as a product of binomial coefficients and powers. Please explain your work.

Spring 2017 COT 3100 Exam #2 - Sheet 3 (3/23/2017)

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6) (10 pts) A class has 13 boys and 17 girls. The class is split into 15 pairs of students. Each boy must be paired up with a girl. (Naturally, a few girls will be pairs with other girls.) In how many different ways can the students be paired up? We consider two ways of pairing up the students as different if at least one pair of students in one of the pairings isn't a pair in the other pairing. Please explain your work.

7) (10 pts) An ant walks on the Cartesian plane, starting at $(0, 0)$ and is traveling to $(10, 20)$. The ant always walks one unit in the positive x -axis or positive y -axis, proceeding between points with integer coordinates.

(a) In how many different ways can the ant make his journey? Please explain your work.

(b) If the ant must pass through location $(4, 17)$, in how many ways can he make his journey? Please explain your work.

8) (5 pts) In the game of chess, played on a 8×8 grid, a rook can attack any square on its row or column. The rows on a chessboard are labeled with the letters A through H and the columns are labeled with the numbers 1 through 8. For example, a rook on D3 can attack any square in row D or column 3. (So, this rook can attack squares D7 and F3, for example.) Consider placing 8 rooks on an otherwise empty chessboard. In how many ways can we place the 8 rooks, so that no rook can attack a square occupied by any other rook? (One such arrangement of rooks would be A1, B2, C3, D4, E5, F6, G7 and H8.) Please explain your work.

Spring 2017 COT 3100 Exam #2 - Sheet 4 (3/23/2017)
Note: This sheet does NOT have a back side with questions!!!!

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Lab Section: 17 18 19 20 21 22 24

9) (14 pts) A new campus restaurant, Universe of Chicken Fingers, offers family meals. A family meal consists of 4 sides and 6 entrées. If Universe of Chicken Fingers offers 10 different side items and 15 different entrees, how many different family meals can be ordered? (Note: Universe of Chicken Fingers has plenty of each item so that one may get two or more orders of the same side item or entrée, if they desire.) Please explain your work.

10) (1 pt) March 23rd is National Chip and Dip Day. What is an ingredient typically used in Bean Dip?
