

Spring 2014 COT 3100 Multiple Choice Exam #2

VERSION A

1) Did you bubble your PID and exam version on your scantron? (Note: You will only get credit for this question if your answer is Yes and if your answer is accurate.)

- a) Yes b) No

2) How many divisors does $2^33^55^2$ have?

- a) 3 b) 10 c) 30 d) 72 e) None of the Above

3) What is the sum of divisors of 2^43^3 ?

- a) 31×40 b) 16×27 c) 32×41 d) 6^7 e) None of the Above

4) Which of the following is NOT a prime number?

- a) 2 b) 17 c) 73 d) 91 e) 103

5) Which of the following is equivalent to $17 \pmod{31}$?

- a) -45 b) -17 c) -79 d) 14 e) None of the Above

6) Let a, b, c and d be positive integers such that $a \mid b$ and $c \mid d$. Which of the following assertions is always true?

- a) $(a + c) \mid (b + d)$ b) $ab \mid cd$ c) $ac \mid bd$ d) $ad \mid bc$ e) None of the Above

7) When dividing 183 by 22, we obtain a quotient q and a remainder r , as defined by the division algorithm. What is $q+r$?

- a) 7 b) 8 c) 15 d) 24 e) None of the Above

8) What is the greatest common divisor of 28 and 182?

- a) 1 b) 2 c) 7 d) 14 e) None of the Above

9) What is the greatest common divisor of 24 and 66?

- a) 2 b) 6 c) 12 d) 24 e) None of the Above

10) What is the least common multiple of 24 and 66?

- a) 66 b) 132 c) 792 d) 1584 e) None of the Above

11) According to the course textbook, all ISBN-10 numbers $d_1d_2\dots d_{10}$ satisfy the following property: $\sum_{i=1}^{10} ix_i \equiv 0 \pmod{11}$. If the first 9 digits of an ISBN-10 number are all 2, what is the last digit?

- a) 2 b) 3 c) 4 d) 5 e) None of the Above

12) Which of the following parts is NOT part of a proof by mathematical induction?

- a) base case b) inductive hypothesis c) counter-example
d) inductive step e) None of the Above

13) Which of the following statements is true?

- a) $\sum_{i=1}^{2(n+1)} f(i) = (\sum_{i=1}^{2n} f(i)) + f(2n + 2)$
b) $\sum_{i=1}^{2(n+1)} f(i) = (\sum_{i=1}^{2n} f(i)) + f(2n + 1) + f(2n + 2)$
c) $\sum_{i=1}^{2(n+1)} f(i) = (\sum_{i=1}^{2n} f(i)) + (\sum_{i=1}^{2n+2} f(i))$
d) $\sum_{i=1}^{2(n+1)} f(i) = (\sum_{i=1}^{2n} f(i)) + (\sum_{i=2n}^{2n+2} f(i))$
e) None of the Above

14) What is the difference between regular mathematical induction and strong induction?

- a) The strong inductive hypothesis assumes that the given formula is true for potentially several different values instead of just one value.
b) Strong induction does not require proving the inductive step.
c) Strong induction does not require proving any base cases.
d) Strong induction requires proving two different inductive steps.
e) None of the Above

15) Consider proving that $5 \mid (n^5 - n)$ using mathematical induction for all non-negative integers. Would the proof involve algebra that required expanding a binomial of the form $(k + 1)^5$?

- a) Yes b) No

16) If I want to prove that $A > B$, which of the following set of steps will prove it?

- a) $A > C > D > E > B$
b) $A > C < D > E > B$
c) $A < C < D < E < B$
d) $A > E > D < C > B$
e) None of the Above

17) Let H_n stand for the n^{th} Harmonic number. What is H_3 ?

- a) 1 b) 1.5 c) 2 d) 3 e) None of the Above

18) Consider the equation $17x \equiv 14 \pmod{44}$. If we know that $17^{-1} \pmod{44}$ equals 13, what value of x satisfies the equation?

- a) 1 b) 3 c) 6 d) 10 e) None of the Above

19) If we want to test whether or not 9973 is prime, what is the largest prime number we have to try to divide into it? (Note: 9973 is very close to 10000.)

- a) 83 b) 97 c) 99 d) 9967 e) None of the Above

20) March 14th is often known as Pi day because 3.14 is an approximation to what well-known mathematical constant?

- a) π b) e c) Avogadro's Number d) i e) ϕ

Spring 2014 COT 3100 Free Response Exam #2

Last Name: _____ , **First Name :** _____

1) (10 pts) Use the Extended Euclidean Algorithm to find $72^{-1} \pmod{239}$. Please give your answer as an integer in between 0 and 238, inclusive. A majority of the grade will be given for your work and not the final answer.

2) (10 pts) Prove or disprove: Let a , b and c be arbitrary positive integers greater than 1. If $\gcd(a, b) = 1$ and $\gcd(b, c) = 1$, then $\gcd(a, c) = 1$.

3) (10 pts) Let H_n denote the n^{th} Harmonic number. Use mathematical induction on n to show that $H_{2^n} \geq 1 + \frac{n}{2}$, for all non-negative integers n .

4) (10 pts) Let $f(n) = \frac{n}{n+2}$. Define $f^k(n)$ to be the function f composed with itself k times. More formally, $f^0(n) = n$ and $f^k(n) = f(f^{k-1}(n))$, for all positive integers k . Using induction on k , prove that for all positive integers k , $f^k(n) = \frac{n}{(2^k-1)n+2^k}$. (Hint: The algebra can be messy if you don't multiply both your numerator and denominator by $(2^k - 1)n + 2^k$. So, in full, after you do a particular step, you would take your fraction and multiply it by $\frac{(2^k-1)n+2^k}{(2^k-1)n+2^k}$. Please feel free to ignore the hint, but I do think it reduces the amount of algebra drastically.