COT 3100H Exam #2 - Part 1 (Number Theory) - 25 pts (10/22/2024)

Last Name: __________________ , First Name: ____________________

1) (6 pts) $n = 6^7 15^4$. How many divisors does n have?

____

2) (8 pts) Although we didn't explain it in class, the prime sieve can work on a range of integers not starting with 1. The trick to making it work is figuring out the first number that you have to cross out in the range, since any one of the first p numbers in an arbitrary range of integers might be divisible by the prime p. Use this technique, determine all of the prime numbers in between 111 and 120. Note that because $120 < 121 = 11^2$, you only have to do four sets of cross offs for the four primes less than 11: 2, 3, 5 and 7. In your work below, write out each integer that is crossed off on each pass. (Note: The same integer might be crossed off more than once.) Then, write down the prime numbers in between 111 and 120, inclusive.

111 112 113 114 115 116 117 118 119 120

Cross Offs (more slots might be provided than necessary for each of the lists below.)

2: _____________ , _____________ , _____________ , _____________ , _____________

3: _____________ , _____________ , _____________ , _____________ , _____________

5: _____________ , _____________ , _____________ , _____________ , _____________

7: _____________ , _____________ , _____________ , _____________ , _____________

Primes in Range: _____________ , _____________ , _____________ , _____________

3) (11 pts) Find all integer solutions (x, y) to the equation $305x + 110y = 35$.

COT 3100H Exam #2 - Part 2 (Induction) - 24 pts (10/22/2024)

4) (12 pts) Prove that for all non-negative integers, n, that $17 \mid (13^{2n} + 4^{2n+2})$.

5) (12 pts) Let F_n denote the n^{th} Fibonacci number. (Recall $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$, for all integers $n > 1$.) Prove that the following summation holds for all positive integers n:

$$\sum_{i=1}^{n} (-1)^{i-1} F_i = (-1)^{n-1} F_{n-1} + 1$$

COT 3100H Exam #2 - Part 3 (More Induction, Rec Probs) - 26 pts (10/22/2024)

6) (8 pts) The arithmetic sequence a_1, a_2, ..., a_{100} has a common difference of 2 and a sum of 350. What is the sum $\sum_{i=1}^{50} a_{2i-1}$ equal to? (Note: This problem can be solved without determining a_1.)

____

7) (6 pts) N and M are positive integers such that $160N = M^4$. What is the minimum possible value of N?

____

8) (10 pts) The sum of the interior angles of a convex n-gon is 180(n-2) degrees. Under the assumption that the sum of the degrees in a triangle is 180 degrees, prove this formula holds via induction on n for all integers $n \geq 3$.

9) (2 pts) In the song Back to December, the event discussed occurred in which month?

____