

COT 3100 Exam #2: Number Theory, Induction, Sheet 1
10/24/2017

Recitation: 18-T12 20-T1 22-T2 19-R12 21-R1 23-R2 24-F3:30

Last Name: _____, **First Name:** _____

1) (8 pts) Convert 6987 in base ten to hexadecimal (base 16).

2) (10 pts) Let a and b be integers such that $7 \mid (2a + 3b)$. Prove that $7 \mid (13a + 16b)$.

3) (15 pts) Determine $59^{-1} \pmod{203}$. Please express your answer as an integer in between 0 and 202. In order to earn full credit you must use the Extended Euclidean Algorithm.

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4) (12 pts) Let $a = 2^3 3^5 5^2 7^1$ and $b = 3^3 5^6 7^2$. How many divisors does a have? How many divisors does b have? Express the greatest common divisor of a and b in prime factorized form. Express the least common multiple of a and b in prime factorized form.

Number of Divisors of a = _____

Number of Divisors of b = _____

GCD(a, b) = _____

LCM(a, b) = _____

5) (15 pts) Using induction on n , prove for all non-negative integers n that $9 \mid (2^{2n} + 6n - 1)$.

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6) (10 pts) Using induction on n , prove for all positive integers n that $\sum_{i=1}^n i^2 \leq n^3$.

7) (16 pts) Let $H_n = \sum_{i=1}^n \frac{1}{i}$. Using induction on n , prove for all positive integers n that

$$\sum_{i=1}^n iH_i = \frac{n(n+1)}{2}H_n - \frac{n(n-1)}{4}$$

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8) (12 pts) Define the sequence t_n as follows: $t_0 = 7$, $t_1 = 10$, $t_n = 3t_{n-1} - 2t_{n-2}$, for all integers $n \geq 2$. Prove, using strong induction on n with 2 base cases, that for all non-negative integers n ,

$$t_n = 4 + 3(2^n).$$

9) (2 pts) How many losses has the undefeated UCF Knights football team suffered in the 2017 season?
