COT 3100 Section 201 Exam #1 (1/31/2024)

Last Name: _____ , First Name: _____

1) (10 pts) Use the laws of logic to show that the two following Boolean expressions are logically equivalent. **Please separate out Commutative and Associative steps.**

(a) $(p \wedge \big(q \vee (r \wedge q)\big)) \vee (q \wedge \overline{(\bar{r} \vee p) \wedge p}$

(b) q

Please use the format shown in class to show the equivalence.

2) (9 pts) The following truth table was correctly filled out by a student, but the column headers (in their place, A, B and C are written), which were supposed to be Boolean expressions were erased.

p	q	r	A	B	C
F	F	F	T	T	T
F	F	T	T	F	F
F	T	F	F	T	F
F	T	T	F	T	F
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	F	T	F
T	T	T	F	T	F

Determine possible expressions for A, B and C, limiting yourself to the variables, p, q and r. In addition to your answers, explain briefly how you came up with them.

Reasoning:

A: _____ B: _____ C : _____

3) (6 pts) Jesse races in the Daytona 500, a 500 mile race. When he is driving, he averages 160 miles per hour. He ends up having to stop for pit stops every 47 miles (first stop is at mile 47, next at mile 94, etc.) Each stop takes 1 minute, 15 seconds. What is his actual average speed over the course of the whole race, from start to finish., in miles per hour?

_____

4) (10 pts) Prove or disprove the following statement for finite sets A, B, C and D:

If $A \subseteq B \cup C$ and $B \subseteq D$, then $A \cap C \subseteq D$.

5) (10 pts) Prove of disprove the following statement for finite sets A, B and C:

If $A \subset B$ and $C \subseteq A$, then $A - C \subset B - C$

6) (10 pts) Each of the 100 students at Hilldale Elementary speak English, French or Spanish. 85 students speak English or French, 70 students speak French or Spanish and 95 students speak English or French. If we tallied 1 point for each student who spoke English, 1 point for each student who spoke French and 1 point for each student who spoke Spanish, 160 total points would be tallied. (Some students contribute 1, others 2 and a few 3, to this tally.) How many students speak all three languages out of the 100 total student? (Most of the credit will be given for the justification via the Inclusion-Exclusion Principle and not the answer.)

7) (7 pts) Solve the following equation for x:

$$log_4 x = log_x 16$$

Express your answer in the form a^b, where a is a positive integer and b is irrational.

---

8) (6 pts) Prove or disprove the following statement for all real numbers x and y:

$$\forall x \exists y \, [\, xy + 11 = \, 4x + 3y \,]$$

9) (6 pts) Jenna drives a motorboat downstream of a river with a current of 4 miles per hour for 2 hours. She decides to return, driving the motorboat upstream for another 2 hours, but only makes it two-thirds of the way back to her original starting point. How far (in miles) is she from her starting point when she stops going upstream?

_____

10) (1 pt) On January 31, 1865, the House of Representatives passed the 13th amendment. How many amendments that were currently in effect had been previously passed by the House of Representatives, prior to the 13th amendment?

_____

Scratch Page – Please clearly mark any work on this page you would like graded.