COT 3100 Section 2 Exam #1 - Part 1 (Logic) - 25 pts (2/2/2023)

Last Name: _____ , First Name: _____

Circle Rec: M8am M4:30pm T10:30am W8am R10:30am R4:30pm F11:30am

1) (8 pts) Fill out the following truth table. Please place a T or F in each empty slot. Any ambiguous letter (according to the grader) will be marked incorrect.

p	q	r	$\overline{p \vee q}$	$\bar{q} \wedge r$	$(\overline{p \vee q}) \vee (\bar{q} \wedge r)$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) (10 pts) Using the laws of logic show that the two following logical expressions are equivalent:

(1) $(\bar{p} \wedge q) \vee (\overline{p \vee q}) \vee (r \wedge \bar{p})$ (2) $\bar{p}$

For the purposes of grading, if you use two rules in one step (such as Commutative with another step), please indicate BOTH reasons on that line.

Step	Reason

3) (7 pts) Prove or disprove the following assertion over the universe of **real (R)** numbers:

$$\forall x \forall y [9x^2 \geq 4y(3x - y)]$$

Please clearly note whether the assertion is true or not, followed a justification of your answer. Most of the points are awarded for the justification.

COT 3100 Section 2 Exam #1 - Part 2 (Sets) - 25 pts (2/2/2023)

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4) (12 pts) Springfield Middle School has three academic teams: the Chess Team, the Math Counts Team and the Debate Team. There are 25 students who are either on the Chess Team **or** Math Counts Team, 10 students who are on both the Chess Team **and** Debate Team, 7 students who are on both the Math Counts Team **and** Debate Team, 5 students are on all three teams, and there are exactly 13 students on the Debate Team. (a) How many students are on **at least** one of the three teams? (b) How many students are on the Debate Team **only**? (Meaning they are on the Debate Team, not on the Math Counts Team, and not on the Chess Team.) Put a box around both answers. (Note: solutions relying on Venn Diagrams will get a maximum of 3 points out of 12.)

5) (8 pts) Prove the following assertion for all sets A, B and C:

If $A \subseteq B$ and $C \subseteq B$, then $(A \cup C) \cap \bar{B} = \emptyset$. **Do NOT use a set membership table.**

6) (5 pts) Disprove the following assertion about sets A, B, C and D by finding a counter-example for which it is false: if $A \subseteq C \cap D$ and $B \subseteq C \cup D$, then $B - A \subseteq C$ or $B - A \subseteq D$. **Explicitly state the elements in sets A, B, C, D and B – A in your counter-example.**

COT 3100 Section 2 Exam #1 - Part 3 (D=rt, logs) - 25 pts (2/2/2023)

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7) (6 pts) Jason ran the first third of a 15 kilometer race at an average speed of 12 km/hr. He ran the rest of the race at an average speed of 6 km/hr. What was his average speed for the whole race? Please answer as a decimal to the nearest tenth in km/hr.

_____

8) (8 pts) Devita biked a race at an average speed of 15 miles/hour, while Kya biked the same race at an average speed of 16 miles hour. If Kya finished the race 20 minutes before Devita (and they started at the same time), how long was the race?

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9) (10 pts) Find the values of x and y, which satisfy the following set of equations. Please simplify your answers to either a single integer or the form $a\sqrt{b}$, where b isn't divisible by any perfect square.

$$log_9(3y^2) = log_{27}(9x^5) \qquad log_9 x + 2log_3 y = 8$$

x = _____ , y = _____

11) (1 pt) Today is Groundhog Day, where many keen observers of the weather anxiously await the sight of Punxsutawney Phil, to see if he sees his shadow. What kind of animal is Phil?

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