Spring 2017 COT 3100 Exam #1 Multiple Choice
Logic, Sets, Number Theory

VERSION A

Please double check that you bubbled your PID and exam version on your scantron. If you do not, you will get a ZERO on this exam.

1) Consider the following argument: "John either owns a yacht or a bicycle. John does not own a yacht. We can conclude that John owns a bicycle." Which rule of inference is being used in deriving the conclusion?

a) Rule of Conjunction   b) Rule of Disjunctive Syllogism

     c) Rule of Conjunctive Simplification   d) Modus Tollens   e) None of the Above

2) Consider the following true statement over the real numbers: $\exists x \forall y [xy = 0]$. Which of the following values can we set x equal to, to prove that this statement is true?

a) 0   b) 1   c) y   d) y^2 e) None of the Above

3) Which of the following is the contrapositive of the Boolean expression $\bar{r} \rightarrow (\bar{p} \vee q)$.

a) $(p \vee \bar{q}) \rightarrow r$ b) $(p \vee \bar{q}) \rightarrow \bar{r}$ c) $(p \wedge \bar{q}) \rightarrow \bar{r}$

     d) $(p \wedge \bar{q}) \rightarrow r$ e) None of the Above

4) Let $P(x, y) = $ "$x + y = 0$". The statement $\forall x \exists y [P(x, y)]$ is a true statement. To prove that this statement is true, for any arbitrary value of x, we must find a value of y which makes the statement true. What is the value of y that makes the statement true?

a) 0   b) 1   c) x   d) x^2 e) None of the Above

5) Let A, B and C be sets with A = {1, 3, 5}, B = {2, 4, 7, 8} and C = {1, 2, 6, 8}. How many elements does $A \times B - C \times C$ contain?

a) 0   b) 2   c) 10   d) 12   e) None of the Above

6) Let A and B be sets such that $|A \cup B| = 46$ and $|A \cap B| = 11$. If A has twice as many elements as B, how many elements does B have?

a) 11 b) 19 c) 38 d) 57 e) None of the Above

7) Let the set A = {1, 3, 4, 5, 7, 10}. How many elements of $\wp(A)$ contain in between one and three odd numbers, inclusive, and no even numbers?

a) 4 b) 15 c) 16 d) 64 e) None of the Above

8) Define a set s(A, B, C), in terms of sets A, B and C as follows:

$$s(A, B, C) = \{x | ((x \in A) \vee (x \in B) \vee (x \in C)) \wedge (x \notin A \cap B) \wedge (x \notin A \cap C) \wedge (x \notin B \cap C)\}.$$

Let A = {1, 2, 5, 9, 12, 17}, B = {2, 3, 9, 15, 20}, and C = {3, 8, 17, 19, 20, 21, 24}. What is $s(A, B, C)$?

a) {1, 2, 3, 5, 8, 9, 12, 15, 17, 19, 20, 21, 24}   b) {4, 6, 7, 10, 11, 13, 14, 16, 18, 22, 23}

c) {1, 5, 8, 12, 15, 19, 21, 24}   d) {2, 3, 9, 17, 20}   e) None of the Above

9) Let a and b be positive integers such that a < b, gcd(a, b) = 6, and lcm(a, b) = 360. What is the maximum possible value of a?

a) 6 b) 12 c) 30 d) 60 e) None of the Above

10) How many divisors does $2^3 3^8 97^2$ have?

a) 13 b) 16 c) 48 d) 108 e) None of the Above

11) How many integer solutions for x and y does $26x + 91y = 1013$ have?

a) 0 b) 1 c) 13 d) infinite e) None of the Above

12) What is the remainder when 103^{105} is divided by 101?

a) 1 b) 2 c) 16 d) 103 e) None of the Above

13) Bruno Mars has won Grammy Awards in the past for his songs "Uptown Funk" and "Just The Way You Are". This year, he won a Grammy as a producer for the Album 25, by Adele. With what planet in our solar system does he share his last name?

a) Mars b) Earth c) Jupiter d) Saturn e) None of the Above

Last Name: _____ , **First Name :** _____

Lab Section: 17   18   19   20   21   22   24

Note: Even though each question is worth 12 points, some of the responses required for full credit may be much shorter or longer than other responses require for full credit. Part of the reason for this is to not give away whether you are to prove or disprove the given claims. (Disproofs tend to be shorter and part of what I am testing is whether or not you can correctly gauge the validity of a claim. I don't want the number of points a question is worth to give away whether the claim is true or false.)

1) (12 pts) Prove or disprove the following assertion for finite sets A, B and C:

$$\text{If } A \cap C = \emptyset, \text{ then } A - (B - C) \subseteq A - B - C$$

2) (12 pts) Prove or disprove the following assertion for finite sets A, B and C:

$$\text{if } A \subseteq B \text{ and } A - C \neq \emptyset, \text{ then } A - B \neq \emptyset.$$

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3) (12 pts) Using the laws of logic, show that the following logical expression with Boolean variables, p, q, and r is a tautology:

$$[(p \wedge r) \vee (p \wedge \bar{r}) \vee (p \wedge q)] \vee [(r \vee \bar{p}) \wedge (\bar{p} \vee \bar{r})]$$

Note: You may not use all of the rows shown below.

Step	Reason
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4) (12 pts) Determine all integer solutions (x, y) to the equation $211x + 45y = 17$.