Spring 2014 COT 3100 Multiple Choice Exam #1 Version A

1) For how many combinations of values of p and q is $p \to q$ true?

a) 0 b) 1 c) 2 d) 3 e) 4

2) What is the inverse of $\bar{q} \to p$? (Note: Only choose an answer that applies the definition of converse to this statement directly. Do NOT, for example, choose the contrapositive of the inverse.)

a) $p \to q$ b) $p \to \bar{q}$ c) $q \to p$ d) $\bar{p} \to \bar{q}$ e) None of the Above

3) Which Logic Law is being applied in the example below?

$$(\bar{r} \wedge q) \vee (\bar{r} \wedge \bar{p}) \leftrightarrow \bar{r} \wedge (q \vee \bar{p})$$

a) Commutative b) Associative c) Distributive d) De Morgan's
e) None of the Above

4) In class we covered a Boolean expression that checked the satisfiability of a Sudoku puzzle. In particular, the variable p(i, j, n) is set to true iff the cell in row i, column j has the value n. Which constraint is checked by the following Boolean expression?

$$\bigwedge_{j=1}^{9} \bigwedge_{n=1}^{9} \bigvee_{i=1}^{9} p(i, j, n)$$

a) Each row contains every number b) Each column contains every number
c) Each 3 x 3 box contains every number d) The sum of each row is 45.
e) None of the above

5) Imagine that your goal is to disprove $\forall x \exists y P(x, y)$. If you could prove which of the following would you successfully disprove the claim?

a) $\forall y \overline{P(7, y)}$ b) $\exists y \overline{P(3, y)}$ c) $\forall y P(5, y)$ d) $\exists y P(4, y)$
e) None of the Above

6) Which of the following is equivalent to $\nexists x P(x)$?

a) $\forall x P(x)$ b) $\forall x \overline{P(x)}$ c) $\exists x P(x)$ d) $\exists x \overline{P(x)}$
e) None of the Above

7) Given the following premises, which of the following conclusions can be drawn with confidence?

i) $p \rightarrow q$ ii) $\bar{r} \rightarrow \bar{q}$ iii) p

a) r b) $\bar{r}$ c) $\bar{q}$ d) $q \rightarrow p$ e) None of the Above

8) Consider the following statement: if a is an odd integer, then (3a + 1)(2a - 2) is divisible by 8. Which proof technique is used below to prove it?

Assume a is odd. Then there exists an integer n such that a = 2n + 1. Substitute into the given expression:

(3a + 1)(2a - 1) = (3(2n+1) + 1)(2(2n+1) - 2) = (6n + 4)(4n) = 8n(3n+2).

Since n is an integer, n(3n + 2) is as well, and the given expression is divisible by 8.

a) Proof by Contradiction b) Proof of the Contrapositive c) Proof of the Inverse
d) Direct Proof e) None of the above.

9) In class you saw a proof that $\sqrt{2}$ is irrational. Which proof technique was used to derive this result?

a) Proof by Contradiction b) Proof of the Contrapositive c) Proof of the Inverse
d) Direct Proof e) None of the above.

10) The statement, "For all positive real values x and y, $\frac{x+y}{2} > \sqrt{xy}$", is false. Which of the following pairs of values for x and y provide a counter-example that disproves this statement?

a) x = 3, y = 5 b) x = 4, y = 4 c) x = 5, y = 3 d) x = 0, y = 0
e) None of the above

11) Which of the following is true, about the following infinite sets defined in the text (and most other math books)?

a) $Z^+ \subseteq N \subseteq R \subseteq Q$ b) $N \subseteq Q \subseteq R \subseteq Z$ c) $N \subseteq Z \subseteq R \subseteq Q$
d) $N \subseteq Z^+ \subseteq R \subseteq Q$ e) None of the Above

12) How many elements are in the Cartesian product of {2, 4, 5, 7, 12} and {a, q, r, z}?

a) 4 b) 5 c) 9 d) 20 e) None of the Above

13) There are 23 students in a class. All of the students either play an instrument or play a sport. If exactly 15 students play a sport and exactly 18 students play an instrument, how many students play both a sport and an instrument?

a) 0 b) 5 c) 8 d) 23 e) None of the Above

14) Let $f(x) = e^{3x+4}$ and $g(x) = \sin(5x - 7)$, which of the following is g(f(x))?

a) $\sin(5(3x + 4) - 7)$ b) $\sin(3^{3x+4})$ c) $\sin(3^{5x-7})$ d) $\sin(5e^{3x+4} - 7)$
e) None of the Above

15) Let $f(x) = \frac{2x-5}{7}$, for a domain of all real x. What is $f^{-1}(x)$?

a) $\frac{7}{2x-5}$ b) $\frac{7}{2}x - \frac{7}{5}$ c) $\frac{7}{2}x + \frac{5}{2}$ d) $7x + \frac{5}{2}$ e) None of the Above

16) Let L_n be sequence defined as follows: $L_1 = 1$, $L_2 = 3$, $L_n = L_{n-1} + L_{n-2}$, for all integers $n > 2$. What is L_7?

a) 18 b) 29 c) 47 d) 76 e) None of the Above

17) What is $\sum_{i=0}^{\infty}(\frac{3}{4})^i$?

a) 1 b) 3 c) 9 d) 16 e) None of the Above

18) If a matrix A has dimensions 2 x 6 and the matrix product AB exists, which of the following are possible dimensions of B?

a) 2 x 2 b) 3 x 2 c) 4 x 6 d) 6 x 6 e) None of the Above

19) A complete undirected graph is one where each pair of distinct vertices has a single edge between it. Consider the adjacency matrix of a complete undirected graph with 6 vertices. How many entries in this adjacency matrix would store a 1?

a) 30 b) 33 c) 35 d) 36 e) None of the Above

20) The non-profit organization (Red) uses its donations to help fight AIDS. What color is its logo?

a) red b) green c) blue d) purple e) yellow

Spring 2014 COT 3100 Free Response Exam #1

Last Name: _____ , First Name : _____

1) (10 pts) Construct a logical expression that has the following truth table. Your logical expression may only use the operators: $\wedge$, $\vee$, and $\bar{}$.

p	*q*	*r*	*Desired Values*
F	F	F	T
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	T
T	T	F	F
T	T	T	T

To prove your answer, construct a truth table with intermediate rows to prove that your expression is equivalent to the one shown above.

Your Expression: _____

p	*q*	*r*				*Your Expression*
F	F	F				T
F	F	T				T
F	T	F				F
F	T	T				T
T	F	F				F
T	F	T				T
T	T	F				F
T	T	T				T

2) (10 pts) Recall that the definition of Cartesian Product of two sets A and B is as follows:

$$A\ x\ B = \{(a, b) | a \in A \wedge b \in B\}$$

For example, if A = {1,3} and B = {2,4,6}, then A x B = {(1, 2),(1, 4),(1, 6),(3, 2),(3, 4),(3, 6)}.

Prove or disprove the following statement for arbitrary sets A, B and C:

if $A \subseteq B$, then $AxC \subseteq BxC$.

3) (10 pts) Use the laws of implication and laws of logic to prove the conclusion shown from the following premises:

$(p \vee q) \rightarrow (r \vee s)$
$(r \vee s) \rightarrow t$
$\bar{t}$
$\bar{p} \rightarrow u$
$\bar{q} \rightarrow v$

$u \wedge v$

Note: You may not use all of the rows shown below.

Step	Reason
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	
15.	
16.	
17.	
18.	
19.	
20.	

4) (10 pts) What is the following sum equal to in terms of n: $\sum_{i=1}^{2n+1}(3i^2)$? Please leave your answer in factorized form.