

Fall 2017 COT 3100 Exam #1 (9/26/2017) (Note: Out of 95 points)

Last Name: _____, First Name : _____

Lab Section: 18-T12 20-T1 22-T2 19-R12 21-R1 23-R2 24-F3:30

1) (8 pts) Complete filling out the truth table below that evaluates the logical expression:

$$(p \wedge \bar{q}) \vee (r \rightarrow \bar{q}).$$

p	q	r	$p \wedge \bar{q}$	$r \rightarrow \bar{q}$	$(p \wedge \bar{q}) \vee (r \rightarrow \bar{q})$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) (12 pts) Using the laws of logic, show that two following expressions are logically equivalent.

$$((\bar{p} \vee r) \wedge (\bar{r} \vee \bar{p})) \rightarrow (\bar{q} \wedge (r \vee \bar{q})) \qquad p \vee \bar{q}$$

Note: You may not use all of the rows shown below.

Step	Reason
1. $((\bar{p} \vee r) \wedge (\bar{r} \vee \bar{p})) \rightarrow (\bar{q} \wedge (r \vee \bar{q}))$	Given
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	
13.	
14.	

3) (12 pts) Use the rules of inference to make the following argument:

p
 $p \rightarrow q$
 $q \rightarrow (r \wedge t)$
 $\bar{r} \vee s$

 s

Step	Reason
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	

4) (10 pts) With proof, prove or disprove the following statement:

$$\forall y \in \mathbb{Z}^+ \exists x \in \mathbb{Q}^+ \left[\frac{x}{y} = x^2 - x \right]$$

Note: The set \mathbb{Z}^+ is the set of positive integers, and the set \mathbb{Q}^+ is the set of positive rational numbers.

5) (8 pts) Prove or disprove: If n is an odd integer, then $4 \mid (n^3 - 1)$.

6) (10 pts) Use the Set Laws to show that the two following sets are equal:

$$A \cup (B \cap \bar{C})$$

$$\overline{\bar{A} \cap \bar{B}} \cap (A \cup (A \cap C) \cup \bar{C})$$

Note: You may not use all of the rows shown below.

Step	Reason
1.	Given
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	

7) (12 pts) Prove or disprove the following assertion about sets A, B and C:

if $B \subseteq A \cap C$, then $A - C \subseteq A - B$.

8) (10 pts) Prove or disprove the following assertion about sets A, B, C and D:

$$(A \cup B) \times (C \cap D) = (A \times C \cap B \times C) \cup (A \times D \cap B \times D)$$

9) (12 pts) Consider finite sets A , B and C where we know the cardinalities of the following sets:

$$|A \cap C| = 7$$

$$|A \cap B \cap C| = 3$$

$$|A \cup B| = 20$$

$$|A \cap B| = 8$$

$$|B \cup C| = 22$$

$$|A \cup B \cup C| = 23$$

Determine $|B|$. (Note: Solutions that only use a Venn-Diagram to determine the answer will receive a maximum of 6/12 points. Only solutions that use a formal proof involving the Inclusion-Exclusion principle will earn full credit.)

10) (1 pt) At what time in the afternoon does one develop a five o'clock shadow? _____

Scratch Page - Please clearly mark any work on this page you would like graded.