COT 3100 Homework #4: Functions, Sequences, Sums, Matrices
Due Date/Time: Friday, January 31st, 2014 in recitation

1) (5 pts) An arithmetic sequence of 40 terms $a_1, a_2, \ldots, a_{40}$ has $a_{17} = 25$ and $a_{33} = 121$. Find the sum of the sequence.

2) (5 pts) A geometric sequence of 20 terms terms $a_1, a_2, \ldots, a_{20}$ has $a_{12} = 42$ and $a_{18} = 336$. Find the sum of the sequence.

3) (10 pts) Let S = $\sum_{i=1}^{\infty}(2i-1)(\frac{2}{3})^i$. Write out the first five terms of S. Use either the subtraction technique or derivative technique shown in class to find the value of S.

4) (10 pts) Let T = $\sum_{i=1}^{20}(2i-1)(\frac{2}{3})^i$. (Essentially, T is the first 20 terms of S.) Use the work in your solution for #3 to determine T. Do not solve for a decimal value of T. Instead, given an expression for T in terms of the various constants involved in the problem.

5) (10 pts) Consider the following recurrence relation: $a_1 = 1$, $a_2 = 11$, $a_n = a_{n-1} + 2a_{n-2}$, for all integers $n > 2$. Prove that the sequence $\{b_n\}$, where $b_n = 2^{n+1} + 3(-1)^n$ satisfies the given recurrence relation. Also show that $b_1 = a_1$ and $b_2 = a_2$. What conclusion can you draw from both of these observations?

6) (5 pts) Let F_n denote the n^{th} Fibonacci number. Prove that $\begin{bmatrix} F_{n+1} \\ F_n \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix}$. Given this result, intuitively explain why $\begin{bmatrix} F_n \\ F_{n-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n-2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

7) (5 pts) Every company makes peculiar restrictions for passwords, but employees of ABC have particularly strange restrictions. Their passwords may only consist of the lowercase letters a, b and c. Furthermore, for any a in a password, it must be followed by a b or c. For any b in a password, it must be followed by an a, and c may be followed by any of the three letters. For examples, abaccba is a valid password of length 7. (Note that for both b's, a comes right after them.) Using a transition matrix, find the number of valid passwords of length 12. You may either use a computer program or a calculator to do the appropriate matrix exponentiation and multiplication. (Extra credit for anyone who can personally email me the full solution at dmarino@cs.ucf.edu before I go over how to solve this question in class.) Note: the work for this question is more difficult than the others but is only worth 5 points because I wanted to pose an interesting question without students' grades suffering too much.