COT 3100 Homework #2: Sets
Due Date/Time: Friday, January 24th, 2014 in recitation

1) Determine all of the elements in the following sets:

a) $\{3^n \mid n \in \mathbf{N}, n < 10\}$
b) $\{4^n - 72(2)^n + 512 = 0 \mid n \in \mathbf{N}\}$
c) $\{n^3 - 4n^2 + 3n \mid n \in \{0,1,2,3,4\}\}$
d) $\{ n \mid n \in \mathbf{N}, n/4 \text{ is an integer}, n < 20 \}$

2) Given that our universe U has 50 elements ($|U| = 50$) and that A, B and C are sets such that $|A| = 24$, $|B| = 24$, $|C| = 24$, $|(A \cap B) \cap C| = 7$, $|A \cap B| = 16$, and $|(A \cup B) \cup C| = 40$, find the following values. Please show your work. (Note: if the values can not be determined given the information above, state this and give two examples (by use of a Venn Diagram) where the size of the set in question is different, but all of the above properties hold.) Hint: You will use the three set version of the Inclusion-Exclusion Principle. It's highly recommended that you prove this to yourself via the two set version. (A proof is included in the posted notes online, but doing this proof on your own provides a valuable learning experience.) You may use a Venn Diagram to provide intuition for your answers, but must prove them formally using the Inclusion-Exclusion Principle.

a) $|U - ((A \cup B) \cup C)|$
b) $|U - (A \cap (B \cap C))|$
c) $|A \cap C|$
d) $|A \cup B|$
e) $|(C - A) - B|$

3) Prove this equality between two sets by using the laws of set theory AND the table method.

$$A \cup ((A \cup C) \cap B \cap (\overline{\overline{A} \cap C})) = A$$

Prove or disprove the assertions given in questions 4, 5 and 6 for arbitrary finite sets A, B and C.

4) If $B \subseteq C$, then $(B - A) \subseteq (C - A)$.

5) If $(B - A) \subseteq (C - A)$, then $B \subseteq C$.

6) If $A \subset C$ and $B \subset C$, then $A \cup B \subset C$.