COT 3100 Fall 2018 Homework #2
Please Consult WebCourses for the due date/time

1) For all integers n, prove that n(3n+1) is an even integer.

2) Let n be an odd integer. Using the result from question #1, prove that $16 \mid (n^4 - 1)$. (Note: this question does require you to multiply out an expression of the form $(x+y)^4$. The algebra is not as bad as it looks…also you can apply the binomial theorem to speed up the work. Google it if you haven't seen it before.)

3) Given a set of n positive real numbers $a_1, a_2, \ldots, a_n$, where $n > 1$, with an average of b, prove that the value of the largest element in the set is strictly less than bn.

4) Let S = {1, 4, 7, 9} and T = {1, 2, 7, 8}. Explicitly list the members of the following sets: $S \cup T$, $S \cap T$. $S - T$, $S \times T$, $T \times S$, $\wp(S)$ and $\wp(T)$.

5) Use set laws to prove that the two following sets are equivalent.

(1) $A \cup B$ (2) $(A \cap B) \cup (A \cap \bar{B}) \cup (\bar{A} \cap B)$

6) Let A, B and C be arbitrary sets taken from the positive integers.

Prove or disprove: If $A \cap B \cap C = \varnothing$, then $(A \subseteq \overline{B}) \vee (A \subseteq \overline{C})$.

7) Give a summary of the life and mathematical contributions of Augustus De Morgan. Please aim for a length of roughly 200 - 400 words. ***Your summary must be typed.*** Please state the sources you used in writing your summary.