

COT 3100 Fall 2024 Homework #6
Please Consult WebCourses for the due date/time

1) (6 pts) Let R_1 and R_2 be relations on a set $A = \{1, 2, 3, 4, 5\}$.

In particular, let $R_1 = \{(1, 3), (1, 4), (2, 2), (2, 5), (3, 1), (4, 2), (4, 5), (5, 1), (5, 3)\}$ and
 $R_2 = \{(1, 5), (2, 1), (2, 4), (3, 1), (3, 2), (4, 5), (5, 2), (5, 3)\}$.

Determine the following:

- a) The relation $R_1 \circ R_2$.
- b) The relation $R_2 \circ R_1$.

2) (5 pts) Let R be a relation over the positive integers defined as follows:

$$R = \{(a, b) \mid \gcd(a, b) > 1\}$$

Determine whether or not R satisfies the following properties. Give a brief justification for each of your answers.

- (i) reflexive
- (ii) irreflexive
- (iii) symmetric
- (iv) anti-symmetric
- (v) transitive

3) (6 pts) How many symmetric relations on the set $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ contain the ordered pairs $(1, 1)$, $(3, 3)$, $(4, 4)$, $(5, 8)$, and $(9, 1)$?

4) (5 pts) How many equivalence relations are there on the set $A = \{1, 2, 3, 4, 5\}$. Note that we can denote an equivalence relation as a set of sets. For example, $\{\{1, 2, 3\}, \{4, 5\}\}$ denotes the equivalence relation where 1, 2 and 3 are all related to each other (and not 4 or 5), and 4 and 5 are related to each other. Note that $\{\{1, 2, 3\}, \{4, 5\}\}$ should count differently than $\{\{1, 2, 4\}, \{3, 5\}\}$. This is a reasonably difficult question that I didn't cover in class, but it's solvable based on the counting techniques that we've learned, so I thought it would be fun to put it on here. You might consider trying to build a recurrence relation to answer this question more generally and use that to build up the answer. I'll write some code also to verify the answer. Feel free to do the same. (I am sure you can get the answer by googling it but that's not too much fun...)

5) (8 pts) Let $f: A \rightarrow B$ and $g: B \rightarrow C$ be functions where A , B and C are finite sets. Given that the function $g \circ f$ is surjective, (a) is f necessarily surjective? (b) is g necessarily surjective? Please answer each question with proof. (If you answer "yes", provide a general proof. If you answer "no", please provide an individual concrete counter-example.)

6) (4 pts) Let $P(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$. $P(2) = P(3) = P(4) = P(5) = P(6) = 0$. What is the value of $a + b + c + d + e$?

7) (4 pts) Let $f(x) = x^2 - 10x + 12$ with a domain of all real $x \in [-\infty, 5]$. Prove that f is injective. What is the range of f ? (You may either use Calculus or complete the square to prove your answers.)

8) (4 pts) Find $f^{-1}(x)$ for the function given in question #7.

9) (8 pts) Let $f(x) = x^3 - 10x^2 + 20x - 30$. Let the roots of $f(x)$ be r, s and t .

(a) Determine the value of the expression $r^2 + s^2 + t^2$.

(b) Determine the value of the expression $\frac{1}{r} + \frac{1}{s} + \frac{1}{t}$.