COT 3100 Fall 2017 Homework 9
Please Consult WebCourses for the due date/time.

1) Let R_1 and R_2 be relations on a set A = {1, 2, 3, 4}.
In particular, let R_1 = {(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (3, 4), (4, 3) (4, 4)} and R_2 = {(1, 2), (2, 3), (3, 4), (1, 3), (2, 4)}.

Determine the following:

a) Whether or not R_1 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
b) Whether or not R_2 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
c) The relation $R_1 \circ R_2$.
d) The relation $R_2 \circ R_1$.
e) $R_1 \cup R_2$
f) $R_1 \cap R_2$
g) The reflexive, symmetric and transitive closures of both R_1 and R_2.

2) Let R be a relation on the set Z^+ defined as follows:

R = {(a, b) | $\exists c \in Fibonacci\ such\ a + b = c\ or\ a - b = c$}

Let the set Fibonacci be the set of positive integers that are Fibonacci numbers.

Determine (with proof) whether or not R is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.

3) Let b(n) equal the number of bits set to 1 in the binary representation of the positive integer n. Prove that the relation, R, defined below over the ***set of positive integers in between 1 and 1024***, inclusive, is an equivalence relation. Into how many equivalence classes does R partition the set described? Explicitly list all of the members of the following equivalence classes: [2] and [520]. Let the set X be the largest of the equivalence classes. What is the smallest integer that belongs to X?

R = {(x, y) | $b(x) = b(y)$}

4) Let R be a relation on the set Z^+ defined as follows:

R = {(a, b) | $|a - b| \leq 3$}

Determine (with proof) whether or not R is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.

5) How many anti-symmetric relations on the set A = {1, 2, 3, 4, 5, 6, 7} contain the ordered pairs (2, 3), (3, 3) and (6, 6)?

6) Let $f(x) = x^2 + 2x - 35$ with a domain of all real $x \in [-\infty, -1]$. Prove that f is injective. What is the range of f? (You may either use Calculus or complete the square to prove your answers.)

7) Find $f^{-1}(x)$ for the function given in question #6.

8) Let A be a set of 12 elements and B be a set of 20 elements. How many functions can be defined with the domain of A and the co-domain of B?

9) Let $f(x) = 5x^2 + 2x - 7$ and $g(x) = 3x + 4$. Determine f(g(x)) and g(f(x)).

10) Let $f(x) = 2x + 3$. Let $f^n(x)$ to be the function f composed with itself n times. (For example, $f^3(x) = f(f(f(x)))$.) Using trial and error, conjecture a guess for $f^n(x)$ and use mathematical induction to prove that guess.

11) Give a summary of the life and mathematical contributions of Leonard Euler. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.