Fall 2016 COT 3100 Section 1 Homework 7
Assigned: 11/21/2016
Due: 12/1/2016 (in lecture)

1) Let R_1 and R_2 be relations on a set A = {1, 2, 3, 4}.
In particular, let R_1 = {(1, 1), (1, 2), (2, 1), (3, 4), (4, 4)} and R_2 = {(1, 2), (2, 3), (3, 4)}.

Determine the following:

a) Whether or not R_1 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
b) Whether or not R_2 is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.
c) The relation $R_1 \circ R_2$.
d) The relation $R_2 \circ R_1$.
e) $R_1 \cup R_2$
f) $R_1 \cap R_2$
g) The reflexive, symmetric and transitive closures of both R_1 and R_2.

2) Let R be a relation on the set Z^+ defined as follows:

R = {(a, b) | $\exists c \in Z^+ \; such \; that \; c^2 = a^2 + b^2$}

Determine (with proof) whether or not R is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.

3) Let b(n) equal the number of bits in the binary representation of the positive integer n. Prove that the relation, R, defined below over the positive integers is an equivalence relation. Into how many equivalence classes does R partition the set of integers? Explicitly list all of the members of the following equivalence classes: [2], [12] and [27]. How many elements are in the equivalence class $[2^k]$ for any positive integer k, in terms of k?

R = {(x, y) | $b(x) = b(y)$}

4) Let R be a relation on the set Z^+ defined as follows:

R = {(a, b) | $|a - b| \le 5$}

Determine (with proof) whether or not R is reflexive, irreflexive, symmetric, anti-symmetric and transitive or not.

5) Let R be a relation on the set $Z^+ \times Z^+$ defined as follows:

$R = \{ ((a,b) , (c,d)) \mid a + b \geq c + d \}$.

Is R a partial-ordering relation? Why or why not?

Define $S = \{ ((a,b) , (c,d)) \mid a + b = c + d \}$ also on the set $Z^+ \times Z^+$. Is S an equivalence relation? Prove your answer.

6) How many anti-symmetric relations on the set A = {1, 2, 3, 4, 5, 6} contain the ordered pairs (2, 2), (3, 4) and (5, 6)?

7) Let $f(x) = x^2 + 4x - 32$ with a domain of all real $x \in [-\infty, -2]$. Prove that f is injective. What is the range of f? (You may either use Calculus or complete the square to prove your answers.)

8) Let A be a set of 10 elements and B be a set of 15 elements. How many functions can be defined with the domain of A and the co-domain of B?

9) Let $f(x) = 3x^3 + 2x - 7$ and $g(x) = 4x - 5$. Determine f(g(x)) and g(f(x)).

10) Find $f^{-1}(x)$ for the function given in question #7.

11) Let $f(x) = 3x + 5$. Let $f^n(x)$ to be the function f composed with itself n times. (For example, $f^3(x) = f(f(f(x)))$.) Using induction on n, prove that $f^n(x) = 3^n x + \frac{5}{2}(3^n - 1)$, for all positive integers n.