COT 3100 Homework #9: Counting
Due Date: Friday, April 4ᵗʰ, in recitation

Note: Please _justify_ your answers and why you use each formula.

1) In alpha testing a new software package, a software engineer finds that the number of defects per 100 lines of code is a random variable X with probability distribution

x	1	2	3	4
Pr(X = x)	0.4	0.3	0.2	0.1

   Find (a) $E(X)$; and (b) $Var(X)$

2) A lottery allows a player to choose 5 values out of 40. The goal of the lottery is to break even exactly, making its cash prizes equal to the amount of money spent by the contestants. Each ticket costs $1 to buy and players receive winnings if they match 3, 4 or all 5 numbers. If the payout for matching all 5 numbers is $250,000 and the payout for matching 4 numbers is $1000, how much should the payout for matching exactly 3 numbers be, to the nearest penny?

3) The probability that it rains during a summer's day in a certain town is 0.2. In this town, the probability that the daily maximum temperature exceeds 25 degrees Celsius is 0.3 when it rains and 0.6 when it does not rain. Given that the maximum daily temperature exceeded 25 degrees Celsius on a particular summer's day, find the probability that it rained on that day.

4) When John throws a stone at a target, the probability that he hits the target is 0.4 He throws a stone 6 times. (a) Find the probability that he hits the target **exactly** 4 times. (b) Find the probability that he hits the target for the first time on his third throw.

5) In Arup's Game of Dice, you roll a fair pair of six-sided dice and record the total. If this total is 11 or 12, you win. If it's 2, you lose. In all other cases, you roll the pair of dice again. If the sum of this second roll exceeds the sum of your first roll, you win! Otherwise you lose. (For example, if you roll a 5 followed by a 6, you win, but if you roll a 10 followed by another 10, you lose.) What is the probability of winning Arup's Game of Dice?

6) There are 792 cards in a Topps Set of baseball cards. Imagine buying a pack of 40 of these cards, each of which are randomly chosen. What is the probability that at least two copies of some card will be in the pack?

7) Suppose that one person in 10,000 people has a rare genetic disease. There is an excellent test for the disease; 99.9% of the people with the disease test positive and only 0.02% of the people who don't have it test positive. What is the probability that someone who tests positive has the disease? What is the probability that someone who tests negative does not have the disease?

8) Suppose we flip a fair coin until it either comes up tails twice in a row of we have flipped it seven times. What is the expected number of times we flip the coin?