COT 3100 Fall 2017 Homework 8
Please Consult WebCourses for the due date/time

Note: Please *justify* your answers and why you use each formula.

1) In alpha testing a new software package, a software engineer finds that the number of defects per 100 lines of code is a random variable X with probability distribution:

x	1	2	3	4
Pr(X = x)	0.3	0.2	0.1	0.4

Find E(X) and Var(X).

2) You decide to start a new lottery. All winning combinations are 5 values chosen from the set {1, 2, 3, ..., 80}. When a player buys a ticket, she can buy a Pick-6, Pick-7 or Pick-8. As the names indicate, if you buy a Pick-k, you choose k numbers out of the 80. You win if each of the numbers from the winning combination are included in your choice of k numbers. The payout for winning is $100,000. How much should you be willing to pay for each ticket (Pick-6, Pick-7 and Pick-8) if you wish to break even (based on expected value)?

3) Two percent of students at a school are on the debate team. Given that a student is on the debate team at the school, the probability that her GPA is above 3.7 is .8. Given that a student is NOT on the debate team at the school, the probability that her GPA is above 3.7 is .3. Given that a student's GPA is above 3.7, what is the probability that she is on the debate team?

4) Sam's probability of getting A's on precisely 2 tests out of 4 is $\frac{8}{27}$. Assuming that Sam's probability of getting an A on each individual test is the same and independent from the other tests, and that her chance of getting an A on an individual test is greater than 50%, what is the probability that Sam gets an A on an individual test?

5) In Arup's Game of Dice 3, you roll a fair pair of six-sided dice and record the total. If this total is 3, 5 or 12, you win. If it's a 2 or 11, you lose. In all other cases, you roll the pair of dice again. If the sum of this second roll equals or exceeds the sum of your first roll, you win! Otherwise you lose. (For example, if you roll a 6 followed by a 6, you win, but if you roll a 10 followed by a 9, you lose.) What is the probability of winning Arup's Game of Dice?

6) Suppose we roll a fair 4 sided die with the numbers [1,4] written on them. After the first die roll we roll the die *k* times where *k* is the number on the first die roll. The number of points you score is the sum of the face-values on all die rolls (including the first). What is the expected number of points you will score?

7) Suppose that one person in 10,000 people has a rare genetic disease. There is an excellent test for the disease; 98.8% of the people with the disease test positive and only 0.4% of the people who don't have it test positive. What is the probability that someone who tests positive has the disease? What is the probability that someone who tests negative does not have the disease?

8) Consider the following algorithm to determine if an array, arr, of size n (indexes 0 through n-1, inclusive) is sorted from smallest to largest:

1) Repeat k times:
    i) Choose a random number, i, in between 0 and n-2, inclusive.
    ii) If arr[i] > arr[i+1], then answer that the array is out of order.
2) Answer that the array is in order.

Consider running this algorithm on the following array of size 10:

i	0	1	2	3	4	5	6	7	8	9
arr[i]	17	8	9	26	29	13	14	18	19	16

If we choose to run the algorithm with k = 5, what is the probability that the algorithm erroneously tells us that the array is sorted? Leave your answer as a fraction in lowest terms.

9) Suppose E and F are events in a sample space and p(E) = 3/4, p(F) = 4/5, and p(F | E) =7/8. Find p(E | F).

10) This question deals with buying and using a pair of thumb drives.

(a) The probability a thumb drive you buy is defective is .002. Assume that the probability one thumb drive is defective does not affect the probability of another one being defective. Given that you've bought two thumb drives, what is the probability that both are defective?

(b) Luckily, both of the thumb drives you've bought work! Each has 1 GB of memory available for you to save files. You have four files you would like to save on the two thumb drives of the following sizes: .7 GB, .4 GB, .3 GB and .3 GB. Unfortunately, you've left your nimwit brother to copy the files from your desktop to the two thumb drives. For each of the four files, he randomly chooses one of the two thumb drives to copy the file, not checking if the copy was successful. (Assume a copy successful as long as the requisite space is available.) What is the probability that all four files get copied successfully? Leave your answer as a fraction in lowest terms.

11) Give a summary of the life and mathematical contributions of Andrew Wiles. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.