COT 3100 Fall 2020 Homework #9
Please Consult WebCourses for the due date/time

1) (5 pts) Two dice are rolled. Both have six sides labeled 1 through 6, inclusive, but the second die is not fair. (The first one is.) The probability of the second die landing on the side with k dots is $\frac{k}{21}$. What is the probability of rolling a sum of 9 when these two dice are rolled?

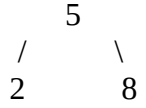
2) (8 pts) Let X be a continuous random variable described below:

$p(X) = x/4$, for all x in the range $0 \leq x \leq 2$
$= x^2/8$, for all x in the range $2 < x \leq k$

(a) What is the value of k?
(b) What is E(X)?

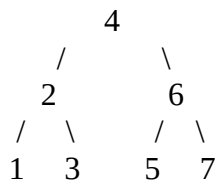
Note: Answers to both questions are irrational numbers.

3) (7 pts) A binary search tree is created by iteratively inserting elements into the tree. To insert an element, into an existing tree, put the element to the left side of the tree if it is less than the value stored at the root (top) of the tree, and put the element to the right side of the tree if it is greater than the value stored at the root of the tree. Repeat this process until there is an opening to insert the item. For example, for the tree shown below:



the value 1 would be inserted to the left of the 2, the values 3 and 4 would be inserted to the right of the 2, the values 6 and 7 would be inserted to the left of the 8 and anything greater than 8 would be inserted to the right of 8, for the next insertion. Where elements end up in the tree depends on the order that they are inserted.

If the values 1, 2, 3, 4, 5, 6, and 7 were inserted in random order into a binary search tree, what is the probability that the final tree structure would look like this:



4) (5 pts) A box contains 5 apples and 6 oranges. Four children each receive a fruit from the box, one after the other, randomly chosen, without replacement. What is the probability that all four children receive the same fruit?

5) (5 pts) A biased coin, when tossed 7 times, has the same probability of obtaining 2 heads out of 7 as it does of obtaining 3 heads out of the 7 tosses. What is the probability the coin lands heads on a single toss?

6) (5 pts) What is the probability that a randomly chosen divisor of 30^{49} is a multiple of 60^{19}?

7) (10 pts) A major TV network would like to maximize the number of games a best of seven sports series is played. Unfortunately, the teams, team A and team B, are not evenly matched. If team A plays at its home stadium, it has a 70% chance of winning. If team A plays at team B's stadium, it still has a 45% chance of winning. Luckily, there is a clause in the contract that the TV network gets to choose which stadium the games are played in, so long as no more than 4 games are scheduled for either stadium. But, the TV network must schedule the location of all games before the series starts. (So, they can't see who actually wins a game before deciding where to schedule the next one.) How should the network schedule the games to maximize the expected number of games played, and what is the expected number of games played for that schedule. Note: a complete schedule is an ordering of seven game locations, where each location is either team A or team B, and there aren't more than 4 As or 4 Bs. For example, the schedule ABBAAAB, represents that we play the first game, fourth game, and if necessary the fifth and sixth games at team A's home stadium. Also note that the teams stop playing games as soon as one team reaches four wins, and that each game always ends in a win for exactly one team. (There are no draws.) **Note: This is a challenging question, don't feel bad if you don't get it!**

8) (5 pts) Please give a summary of the life and mathematical contributions of Pafnuty Chebyshev. Please aim for a length of roughly 200 - 400 words. ***Your summary must be typed.*** Please state the sources you used in writing your summary.