Fall 2016 COT 3100 Section 1 Homework 3
Assigned: 9/20/2016
Due: 9/30/2016

1) Prove or disprove: if the quotient (as defined by the division algorithm) when dividing a_1 by b_1 is q_1 and the quotient when dividing a_2 by b_2 is q_2, then the quotient when dividing a_1a_2 by b_1b_2 is q_1q_2.

2) Convert each of the following numbers in base 10 to the designated base.

a) 12345 to base 8
b) 54321 to base 16
c) 9999 to base 7
d) 7364 to base 5
e) 1024 to base 2

3) Looking at the Fibonacci number recurrence and Euclid's algorithm, explain why the greatest common divisor of two consecutive Fibonacci numbers is always 1. (If you don't know what the Fibonacci numbers are, Google them or look them up in the recommended text.)

4) Determine the greatest common divisor between the following pairs of integers:

a) 123 and 67
b) 871 and 546
c) 609 and 377
d) 399 and 138

5) Prove the following for positive integers, a, b, c and n:

If $an \equiv b(modc)$, then $\frac{an}{\gcd(a,c)} \equiv \frac{b}{\gcd(a,c)} (mod \frac{c}{\gcd(a,c)})$

6) Determine 73^{-1} mod 129 via the Extended Euclidean Algorithm.

7) Let a and n be relatively prime positive integers. (Thus, gcd(a, n) = 1.) Consider the set S = $\{ai | i \in Z, 0 \leq i < n\}$. Prove that each value in S is unique mod n. You may use the following theorem in your proof: If x | (yz), and gcd(x, y) = 1, then x | z. (Hint: Use proof by contradiction and assume that two distinct values in the set are equivalent mod n. If two values are equivalent mod n, then their difference is divisible by n.)

8) Let a be an integer such that $a \equiv 1 \pmod 3$. Prove that $a^3 \equiv 1 \pmod 9$.

9) Using the result from (a) and the fact that if $x \equiv 1 \pmod m$, $x \equiv 1 \pmod n$, and $\gcd(m,n) = 1$, then $x \equiv 1 \bmod (mn)$, prove that

$$666666666667^3 \equiv 1 \pmod{18}.$$

(Note: You may assume that an odd number cubed is odd.)

10) Recount a paragraph or so about the contributions of Srinivasa Ramanujan, related to computer science. (You may do your research from anywhere, but please do not plagiarize. Write things in your own words.)