COT 3100 Fall 2018 Homework #4
Please Consult WebCourses for the due date/time

1) Find all integer solutions to the equation $135x + 198y = 39$.

2) (a)Find all integer solutions to the equation $255x + 104y = 1$.

(b) Find all integer solutions to the equation $255x + 104y = 13$.

(c) Find $104^{-1} \bmod 255$. (Note: Answer must be in between 0 and 254, inclusive.)

3) Let x and y be integers such that $13 \mid (6x - y)$. Prove that $13 \mid (5x + 23y)$.

4) On the first exam, you were asked to prove the following statement:

If n is an integer such that $n \equiv 1 \pmod 6$, then prove that $n^2 \equiv 1 \pmod{24}$.

It turns out that this statement is a specific version of a more general statement that can be proven, which is as follows:

If n is an integer and k is a non-negative integer prove that:

$$\text{If } n \equiv 1 \pmod{4k+2}, \text{ then } n^2 \equiv 1 \pmod{(16k+8)}.$$

5) Let $a = 2^3 3^4 5^2 7^9$, $b = 2^2 3^9 5^4 11^8$, and $c = 2^4 3^6 5^5 11^4$. Determine, in prime factorized form, both gcd(a, b, c) and lcm(a, b, c).

6) For the numbers a, b and c listed in problem 5, determine the number of divisors each of those numbers has.

7) Without prime factorization, calculate the least common multiple of 135 and 198.

8) Give a summary of the life and mathematical contributions of Leonard Euler. Please aim for a length of roughly 200 - 400 words. ***Your summary must be typed.*** Please state the sources you used in writing your summary. (Note: Euler is my favorite mathematician!)