

COT 3100 Fall 2017 Homework #3
Please Consult WebCourses for the due date/time

1) In general, you were told in class that for all integers a and b and positive integers n , if $a \equiv b \pmod{n}$, then $f(a) \equiv f(b) \pmod{n}$, where f is any function that operates on integers only. Using the definition of mod only, prove this specifically for the function $f(a) = a^2$.

2) Convert the following values from the bases indicated to base 10:

i) 3645_7

iv) 30020_4

ii) $AAF7_{16}$

v) 101101011001_2

iii) 12345_9

3) Convert the following base 10 values to the bases indicated:

i) 12435 to base 12

iv) 4921 to base 7

ii) 79770 to base 16

v) 88264 to base 8

iii) 691 to base 2

4) Prove that if n is an integer, then $n(3n+1)$ is an even integer.

5) Prove that if n is an odd integer, then $n^4 \equiv 1 \pmod{16}$. You may use the result from problem 4 to aid you in this proof. (Hint: At some point when you do your algebra, you should get an expression of the form $a(3a + 1)$ where a is an integer. It is extremely helpful when you get to this point to use the result proved in question 4.)

6) Let x and y be integers such that $12 \mid (3x + 4y)$. Prove that $12 \mid (21x + 16y)$.

7) Give a summary of the life and mathematical contributions of Evariste Galois. Please aim for a length of roughly 200 - 400 words. **Your summary must be typed.** Please state the sources you used in writing your summary.