COT 3100 Homework #1: Logic
Due Date/Time: Friday, January 17th in recitation

1) Construct a truth table for the following logical expression: $p \rightarrow (\bar{q} \vee r)$.

2) Construct a truth table for the following logical expression: $(\bar{p} \leftrightarrow q) \rightarrow (r \rightarrow p)$.

3) When planning a party you want to know whom to invite. Among the people you would like to invite are three touchy friends. You know that if Jasmine attends, she will become unhappy if Samir is there, Samir will attending only if Kanti will be there also, and Kanti will not attend unless Jasmine does also. Which combinations of these three friends can you invite so as not to make someone unhappy? Assign logical variables to statements that are part of the problem and use the appropriate formal construct to justify your answer.

4) Using logic laws, show that the following are logically equivalent:

$(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$

5) Using only the laws of logic, show that following two expressions are equivalent:

a) $\left((p \vee \bar{p}) \wedge (q \vee \overline{\bar{q} \vee r})\right) \vee ((p \vee \bar{p}) \wedge q)$
b) q

6) Prove or disprove the following statements. Assume that the domain of each variable presented is the set of integers.

a) $\forall x[\exists y, z \,|x = yz]$
b) $\exists x[\forall y|xy = 0]$
c) $\exists x[\forall y|xy = 1]$
d) $\forall x[(x > 0) \rightarrow (x^2 + x - 6 > 0)]$
e) $\exists x[(x > 2) \wedge (x^2 + x - 6 < 0)]$

7) Establish the validity of the following argument. Clearly list each step and the rule you have used.

$$p$$
$$p \to t$$
$$t \to q$$
$$r \to s$$
$$\neg q \vee \neg s$$

$$\therefore \neg r$$

8) Use the Laws of Logic and Rules of Inference to justify the following argument:

$$(\neg p \vee \neg q) \to (s \wedge r)$$
$$s \to t$$
$$\neg t$$

$$\therefore p$$

9) Use proof by contradiction to show the following:

If a is an arbitrary integer and 3a+7 is even, then a is odd.

10) Use direct proof to show that if a is odd, then 3a+7 is even. Given the results from question 9 and this question, are the two following statements logically equivalent: “a is an odd integer” and “a is an integer and 3a+7 is even an even integer”?