Fall 2024 COT 3100 Homework #1

1) (6 pts) Jennifer is training for a marathon and on three successive days she runs distances of D miles, 2D miles and 3D miles, respectively. Surprisingly, her average speed on those runs got better each day. Namely, on the first day she ran 5 miles/hour, on the second day she ran 6 miles/hour and on the last day she ran an amazing 8 miles/hour. What was her average speed over all three of the runs?

2) (5 pts) Find the values of each of the following bitwise operations. Please show your bit-by-bit work for each calculation.

(a) 27 & 13
(b) 45 | 42
(c) 53 ^ 24
(d) 187 >> 3
(e) 17 << 2

3) (8 pts) Create a truth table for the following logical expression: $\overline{p \wedge q} \rightarrow \bar{r}$. Please make sure your first three columns are for p, q, and r, respectively, that you have appropriate intermediary columns, and that you list your rows in lexicographical order where False comes before True. (This is equivalent to numerical order in binary where we treat False = 0, True = 1 where p is the most significant bit.)

4) (4 pts) Use the truth table method to prove the following two expressions are logically equivalent:

$$p \vee [(p \wedge \bar{r}) \vee (q \wedge p)]$$
$$p$$

5) (7 pts) Use the laws of logic to show that two following expressions are logically equivalent:

$$p \vee [(p \wedge \bar{r}) \vee (q \wedge p)]$$
$$p$$

6) (10 pts) Use the rules of inference to prove the following argument. Make sure to show EVERY STEP, even steps like Disjunctive Amplification.

$p \rightarrow q$
$r \rightarrow s$
$t \rightarrow (\bar{q} \vee \bar{s})$
$(x \vee y) \rightarrow t$
$r \wedge x$

$\therefore \bar{p}$

7) (5 pts) Prove or disprove the following claim over the universe of all **positive** real numbers for x and y.

$$\forall x \left[\exists y \left[\frac{(y - x)}{xy} = \frac{5}{6x} \right] \right]$$

If the claim is false, find a value of x for which it is false. If it is true, show which value(s) of y exist to make the claim true for all x.

8) (5 pts) Jessie, your boss, has chosen two positive real numbers, x and y with x > y. She will either pay you

(a) 4x + y dollars or (b) $4\sqrt{xy}$ dollars

With proof determine which option is better. (Namely, prove that no matter what is chosen for x and y that the option you have chosen is strictly worth more than the other option.)

Note: One way to prove that a quantity A is always greater than a quantity B is to prove that the quantity A – B is always positive.