Fall 2016 COT 3100 Section 1 Homework 2
Assigned: 8/30/2016
Due: 9/9/2016

1) From the premises shown below, prove the conclusion shown. Afterwards, plug in phrases for p, q, r and s that correspond to a natural argument.

$$p \to q$$
$$q \to r$$
$$(q \wedge r) \to s$$
$$p$$

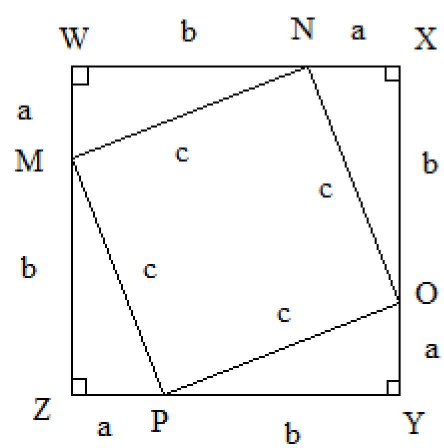
$$s$$

2) Consider a proof that begins, "Let n be an arbitrary positive integer that isn't a perfect square", and ends, "Thus, n has an even number of factors." Which of the four following rules of inference for quantifiers is such a proof using: universal instantiation, universal generalization, existential instantiation, or existential generalization?

3) Using proof by cases, show that for all integers n, $n(n+1)$ is even.

4) Using proof by contradiction, show that for all integers n, $n(n+1)$ is even.

5) Using the diagram shown below, use direct proof to prove the Pythagorean Theorem. (Hint: describe the area of the full figure in two different ways.) In the diagram, the vertices WXYZ determine a square with vertices M, N, O, P on the sides WZ, WX, XY, and YZ, respectively. The triangle MWN is a right triangle with side lengths a and b, and hypotenuse c. Three other triangles congruent to this one appear in the figure.



6) Google the terms "geometric mean" and "harmonic mean" to find their definitions. Prove that the geometric mean of two real numbers a and b, is greater than or equal to the harmonic mean of a and b.

7) Prove the following proposition about arbitrary sets A, B, C, D and E:

If $A \cup B \subseteq C$ and $C \subseteq D \cap E$, then $A \subseteq D \land A \subseteq E$.

8) Disprove the following proposition about arbitrary sets A, B, C, D and E via counter-example:

If $A \cap B \subseteq C$ and $C \subseteq D \cup E$, then $A \subseteq D \lor A \subseteq E$.

9) List out each element in the set $\wp(\emptyset)$. (This is read as the power set of the empty set.) How many elements are in this set?

10) In class, a Cartesian Product was used to describe a possible set of value meals. Come up with a different analogy to describe a Cartesian Product.