COT 3100 Homework #7: Mathematical Induction
Due Date: Friday, March 14th, in recitation

1) Use mathematical induction on n to prove that $\sum_{i=1}^{n} \frac{1}{i^2} < 2 - \frac{1}{n}$, for all integers n > 1.

2) Use mathematical induction on n to prove that $\sum_{i=1}^{n} iH_i = \frac{n(n+1)}{2} H_n - \frac{(n-1)n}{4}$, for all positive integers n. Note that $H_n = \sum_{i=1}^{n} \frac{1}{i}$.

3) Define a sequence of numbers, W, as follows: $W_0 = 2$, $W_1 = 1$ and $W_n = W_{n-1} + \frac{W_{n-2}}{2}$ for all integers *n* > 1. For all positive integers n, prove that $\sum_{i=1}^{n} \frac{W_{i-1}}{W_i W_{i+1}} = 2 - \frac{2}{W_{n+1}}$.

4) Whenever Binary Billy acts up, his punishment is to write binary numbers on the board. He always starts writing 0, 1, 10, 11, 100, etc. Depending on the severity of behavior, Billy has to write all the binary numbers starting at 0 upto all binary numbers with a certain number of digits. For example, if Billy's bad behavior was rated at a 5, then Billy would have to write all the binary numbers from 0 through 11111. Let B(n) denote the total number of binary *digits* Billy must write for a bad behavior rating of n. Using induction on n, prove that $B(n) = (n-1)2^n + 2$, for all positive integers n.

5) There are n cars on a circular track, and amongst them there is enough gas for 1 car to make a complete loop around the track. Show that there is 1 car that can make it completely around the track by pooling gas from every car that it passes by. Though other techniques than mathematical induction can be used to prove this, please use induction to do so. If you can't come up with an inductive proof, but can provide another one, please do so for partial credit.