COT 3100 Homework #6: Mathematical Induction
Due Date: Friday, February 28th, in recitation

1) Use mathematical induction to prove that $\sum_{i=1}^{n}(i \times i!) = (n+1)! - 1$, for all positive integers n.

2) Use mathematical induction to prove that $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} F_{n+1} & F_n \\ F_n & F_{n-1} \end{bmatrix}$, for all positive integers n, where F_k represents the k^{th} Fibonacci number.

3) For all positive integers n, use mathematical induction to prove that $21 \mid (4^{n+1} + 5^{2n-1})$.

4) Use mathematical induction to prove that $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$, for all positive integers n. Note: The trigonometry formulas you may find useful are as follows:

$\cos(A + B) = (\cos A)(\cos B) - (\sin A)(\sin B)$
$\sin(A + B) = (\sin A)(\cos B) + (\cos A)(\sin B)$

This statement is true more generally (for non-integral values of n also) and is known as DeMoivre's Theorem.

5) Use mathematical induction to prove that $\sum_{i=1}^{n}(i(i+1)(i+2)) = \frac{n(n+1)(n+2)(n+3)}{4}$, for all positive integers n.

6) (Extra Credit) The sum shown in #5 is a specific instance of the more general following result:

$$\sum_{i=1}^{n}\left(\prod_{j=i}^{i+k} j\right) = \frac{\prod_{i=n}^{n+k+1} i}{k+2}$$

for all positive integers n and non-negative integers k. Use induction on n to prove this result. (In doing so, treat k as an arbitrary fixed non-negative integer.)