COT 3100 Fall 2024 Homework #4
Please Consult WebCourses for the due date/time

1) Determine the following summation in terms of n: $\sum_{i=1}^{2^n}(i^2 - i)$. (Note: Please express your answer in the form $\frac{2^a(2^b-1)}{c}$, where a and b are functions in terms of n and c is an integer.)

2) Determine the following infinite summation: $\sum_{i=0}^{\infty}(2i + 1)(\frac{1}{3})^i$.

3) Let g(n) be defined as follows be a function defined on the positive integers as follows:

> g(1) = 1, g(2) = 5, g(3) = 6
> For all n > 3, g(n) = 2g(n-1) + g(n-2) - 2g(n-3).

What are the values of g(4), g(5) and g(6)?

If you would like for fun, write a computer program which prints out the first 1000 values of g(n) mod 10^9+7. Feel free to include the source code inside the document containing your homework solutions. If you want to have LOTS of fun, write a program that reads in a value of n upto 10^{12}, and quickly produces the value of g(n) when divided by 10^9 + 7. (Hint: For the latter task, you have to embed the recurrence in a matrix and code up fast matrix exponentiation, which is similar to the fast modular exponentiation taught in class.) Attach your program separately as a .c, .cpp, .py or .java file. Finally, see if you can come up with a closed form formula for g(n)!

4) Prove by induction on *n* that, for all positive integers n:

$$\sum_{i=1}^{n} i3^i = \frac{3}{4}[(2n-1)3^n + 1]$$

5) Prove using induction on n, for all non-negative integers n, that 64 | (9^n - 8n - 1).

6) **Using strong induction on n with three base cases**, prove that a square can be partitioned into n squares, for all positive integers n ≥ 6. (Note: To partition a square, you must draw some line segments dividing the square so that all of the separate pieces are non-intersecting, cover the whole square, and are all squares themselves.) Your proof should primarily have pictures accompanied with the specific dimensions of each of the smaller squares in terms of the square to be partitioned.

7) Prove using induction on n, for all positive integers n, $\sum_{i=1}^{2^n} log_2 i \le (n-1)2^n + 1$.

8) Prove using induction on n, for all non-negative integers n, $\begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}$.