UNIVERSITY OF CENTRAL FLORIDA

Department of Mathematics/School of Electrical Engineering and Computer Science

MAP/COP 3930H Spring 2013

Mathematical Modeling & Scientific Computing

TEST # 2 (Final Exam)

Instructions:

1. Use paper size (8.5 x 11) and write on one side only.
2. Show all relevant steps in arriving at your answer (or conclusion).
3. Clearly designate your answers (circle, underline, etc.)

Part A: Each question carries 12 points.

1. Suppose a student carrying a flu virus returns to an isolated college campus of 1000 students. If it is assumed that the rate at which the virus spreads is proportional not only to the number x of infected students but also to the number of students not infected, determine the number of infected students after 6 days if it is further observed that after 4 days $x(4) = 50$.

2. Given the differential equation

$$\frac{dy}{dx} = (y-1)(y-2)(y-3).$$

   (a) Identify the equilibrium values. Which are stable and which are unstable?
   (b) Construct a phase line. Identify the signs of y' & y''.
   (c) Sketch several solution curves (at least one in each sub-internal of y).

3. Given the first order autonomous system of differential equations

$$\frac{dx}{dt} = -y+1 \quad \text{and}$$

$$\frac{dy}{dt} = x-2.$$

   (a) Find the rest point (or points). Is (are) the rest point (points) stable or (and) unstable.
   (b) Find an equation to the solution curve (That is, the trajectory, path or orbit of the system).
   (c) Sketch the orbit of the system.

4. Solve the initial value Problem

$$y^{-2}\frac{dx}{dy} = \frac{e^x}{e^{2x}+1}, \quad y(0) = 1.$$

5. The population $P(t)$ at time t in a suburb of a large city is governed by the initial-value problem

$$\frac{dP}{dt} = P(10^{-1} - 10^{-7}P), \quad P(0) = 5000,$$

where t is measured in months. What is the limiting value of the population? At what time will the population be equal to one-half of this limiting value?

Part B: Each question carries 10 points.

1. Let $n = 2^{34}5^{13}$. Using the solution outlined in class, determine the number of divisors of n^2 that do NOT divide evenly into n but are less than n. After giving your solution, discuss an alternative method one might have tried that might have been more difficult to use. Explain why one would first pursue this alternative method before trying the more elegant one shown in class.

2. Consider the following game: Player A chooses a number, either 0 or 1 without any knowledge of Player B. Player B does the same. Player A gets paid as defined by the following matrix:

A's payoff matrix	B chooses 0	B chooses 1
A chooses 0	\$9	\$2
A chooses 1	\$2	\$11

Player B gets paid as defined by the following matrix:

B's payoff matrix	A chooses 0	A chooses 1
B chooses 0	\$9	\$3
B chooses 1	\$3	\$8

a) Does A have a dominating strategy? If so, why and what is it? If not, why not?
b) Does B have a dominating strategy? If so, why and what is it? If not, why not?
c) If A and B have an opportunity to meet before playing the game and they agree to split their total earnings, what strategy is their optimal one?

3. Based on the lecture given on Excellence and Creativity, if you were in charge of a start-up video game company, what sorts of decisions would you make to increase the chance that your employees come up with successful games? Give at least one idea that wasn't specifically given in the lecture that is consistent with the principles stated in the lecture.

4. Explain the "curse of dimensionality." Give an example separate to the one given in class that suffers from this problem.