COP/MAP 3930H – Honors Seminar Mathematical Modeling Exam #1

Directions: Write your responses to each question on your own lined paper. Please clearly mark which question number you're answering and staple the pages in order. You may write on both the front and back of each sheet of paper.

1) (20 pts) Find the number of non-negative integer solutions to the equation $a + b + c + d + e = 30$, with $a < 5$, $b < 3$ and $c > 7$. (Hint: Don't forget to use the inclusion-exclusion principle!!!)

2) (15 pts) Let $T(1) = -1$, $T(2) = 7$, and $T(n) = 5T(n - 1) – 6T(n – 2)$ for all integers $n > 2$. Use either method shown in class to find a closed form solution for $T(n)$.

3) (45 pts) Consider a bag with n marbles, with $n > 3$, where 3 of the marbles are red and the rest are blue. A single turn consists of randomly choosing a marble from the bag and replacing it with a blue marble. (Thus, after the first turn, the bad will either have 3 red marbles or 2 red marbles.) Consider repeating turns until the bag only has blue marbles. What is the expected number of turns that will be taken in this process?

Since this is a difficult question, you will be given a set of guiding questions to help you answer the overall question. Though the problem can be solved in other ways, you'll be graded on the specific route of solution I have chosen, dictated by the following smaller questions.

a) (5 pts) If a marble has r red marbles and b blue marbles, what is the probability that a single randomly chosen marble is red?

b) (15 pts) Let the probability found in part a equal p. In this part we would like to derive the expected number of turns, given that we have a probability of p of drawing a red marble on a single turn, until we draw our first red marble. You may solve this part in one of two ways:

> i) Let x be the desired expectation. Set up an equation for x that involves both p and x. This equation will be "recursive", since x will be involved on both sides of it. After setting up the equation, solve for x in terms of p.
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> ii) Write an infinite sum equal to the desired expectation. This sum should be in terms of p. Solve the summation.

c) (5 pts) The linearity of expectation says that if we want to run some process X followed by a process Y, then the expected about of time both processes will take to finish in sequence is simply the expected amount of time for X plus the expected amount of time for Y. Put more mathematically, if X and Y are independent discrete random variables, then $E(X+Y) = E(X) + E(Y)$. We can apply the linearity of expectation to this problem by simply adding up the expectations of the number of turns to remove the first red marble, followed by the second one, followed by the third one. Use this idea to give a final solution to the problem.

d) (20 pts) Write a function trial(int r, int b), which takes in the number of red and blue marbles respectively in the bag and simulates taking turns until the bag has all blue marbles. Have the function return the number of turns the simulation took. (Use Python, C or Java.)

4) (10 pts) Run the Traditional Marriage Algorithm for these individuals, given the ranked lists shown below. Determine who ends up with whom. Each list given is from most desired mate to least desired mate. (For example, Adam would most prefer to be married to Geena.)

Boys: Adam, Bobby, Cam, Dave, Eddie
Girls: Francesca, Geena, Helen, Ivana, Joanna

Adam's List: Geena, Ivana, Joanna, Francesca, Helen
Bobby's List: Joanna, Ivana, Helen, Geena, Francesca
Cam's List: Geena, Joanna, Francesca, Helen, Ivana
Dave's List: Ivana, Geena, Helen, Joanna, Francesca
Eddie's List: Ivana, Geena, Helen, Joanna, Francesca

Francesca's List: Cam, Dave, Eddie, Bobby, Adam
Geena's List: Eddie, Dave, Cam, Bobby, Adam
Helen's List: Cam, Adam, Dave, Eddie, Bobby
Ivana's List: Dave, Adam, Eddie, Bobby, Cam
Joanna's List: Adam, Bobby, Eddie, Cam, Dave

5) (10 pts) Give an anecdote you find telling about a famous mathematician that ***I didn't discuss in class***. State who the mathematician is, one mathematical idea attributed to them, your anecdote, and why you think that anecdote sheds some light about the mathematician's personality. (Note: If you use a story I told in class, then you'll get up to half credit on this question.)