COP/MAP 3930H - Honors Seminar Mathematical Modeling Exam #1

Directions: Write your response to each question on your own lined paper. Please clearly mark which question number you're answering and staple the pages in order. You may write on both the front and back of each sheet of paper.

1) (20 pts) Find a closed form solution for the following recurrence relation:

$$a_n = 2\sqrt{3}a_{n-1} - 4a_{n-2}, \text{ for } n \geq 2.\ a_0 = 1,\ a_1 = 2 + \sqrt{3}$$

2) (20 pts) A bag contains b blue marbles and g green marbles, with $b > g$. Consider randomly pulling each marble from the bag, one by one, in order. What is the probability that you never pull two green marbles in a row? (Hint: consider counting the number of strings of b *Bs* and g *Gs* that don't contain consecutive *Gs*.)

3) (20 pts) Write a program or function in Python, C or Java that runs a million simulations of the situation above with $b = 10$ and $g = 7$ and reports the experimental probability of not pulling two greens in a row. Don't worry about minor syntax issues or efficiency. Focus on creating an accurate algorithm for simulating the experiment described.

4) (20 pts) Sarah is interested in her winning streak in the Pokemon game she plays periodically. She simply records if her current win streak is 0 games, 1 game, or more than 1 game after each game. Given that she lost her last game, her chance of winning the next game is 60%. If she's one exactly one previous game, her chance of winning is 40%. If she's won two or more previous games, her chance of winning again is 30%.

(a) Draw a Markov Chain with three states that encapsulates the situation given.

(b) Using any technique, determine the probability that her current win streak is 0 games, 1 game or more than 1 game. To earn full credit, express each probability as a fraction in lowest terms.

5) (19 pts) Give a detailed summary of either (a) Traditional Marriage Algorithm, or (b) Game Theory Lecture.

6) (1 pt) One's "Erdos Number" refers to the length of the chain of a statement of the form, "I authored a paper with A_1 who authored a paper with ... A_{n-1} who authored a paper with the mathematician X. Who is X?