

Computer Science II

Spring 2018

Final Exam

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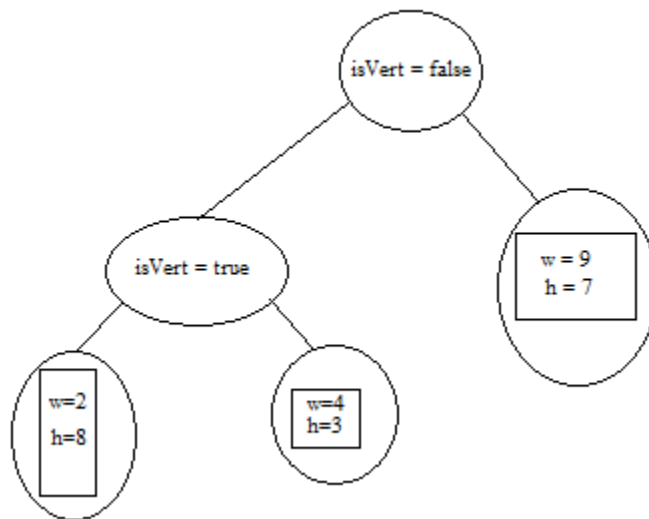
Date: 5/1/2018

Name: _____

1) (15 pts) In two guest lectures, Dr. Ewetz described an open problem dealing with rectangle packing. A single packing operation combines two rectangles with dimensions $w_1 \times h_1$ and $w_2 \times h_2$. We can choose to combine these two rectangles in two ways:

1. Stack on top of each other to create a rectangle of size $\max(w_1, w_2) \times (h_1 + h_2)$
2. Stack next to each other to create a rectangle of size $(w_1 + w_2) \times \max(h_1, h_2)$

Furthermore, for a particular arrangement of rectangles, we can store it in a tree structure, where leaf nodes are the original rectangles and internal nodes represent combining two rectangles in one of these two fashions. Consider the following tree:



In this tree, the non-root internal node represents packing a 2 x 8 and 4 x 3 rectangle vertically, which will yield a 4 x 11 rectangle. Then, the root node, represents packing a 4 x 11 and 9 x 7 rectangle horizontally to yield a 13 x 11 rectangle.

For this problem, you will write a method `getDim()` in the rectangle class that will return an integer array of size 2 storing the width and height, respectively, of the rectangle object (a node in this tree) that the method is called on. The class has the following instance variables:

```
private int w;  
private int h;  
private boolean isLeaf;  
private boolean isVert;  
private rectangle left;  
private rectangle right;
```

If the object represents a leaf node, then `w` and `h` will store the width and height of the rectangle, respectively and `isLeaf` will be set to true. If the object represents an internal node, then `isLeaf` will be set to false, `isVert` will be set to whether or not the two subtree rectangle should be packed vertically or not (true for vertical), and `left` and `right` will point to the left and right subtrees, respectively.

Complete the code for the getDim() method in this class. Note: You may call the method Math.max, which takes in two integers and returns the larger of the two.

```
public int[] getDim() {
    if ( _____ ) return new int[]{w,h};
    int[] leftdim = _____;
    int[] rightdim = _____;
    if ( _____ )
        return new int[]{_____, _____};
    else
        return new int[]{_____, _____};
    }
}
```

2) (10 pts) Below is a 6 line code segment in Java that prompts the user to enter the distance and average speed of a trip and then calculates the time the trip took. For these lines of code, find each direct dependency (ie line 3 must go before line 5) necessary for the code to run properly. Draw the corresponding graph using each line as a vertex and each dependency as an edge. Then, count the number of possible topological sorts of this graph and explain the significance of this number.

```
lemons = int(input("How many lemons?\n"))           # line 1
sugar = int(input("How many cups of sugar?\n"))     # line 2
limitLemons = lemons//3                            # line 3
limitSugar = sugar*4                                # line 4
pitchers = min(limitLemons, limitSugar)             # line 5
print("You can make",pitchers,"pitchers of lemonade.") # line 6
```

Graph

Number of Top Sorts: _____

Significance of # of top sorts: _____

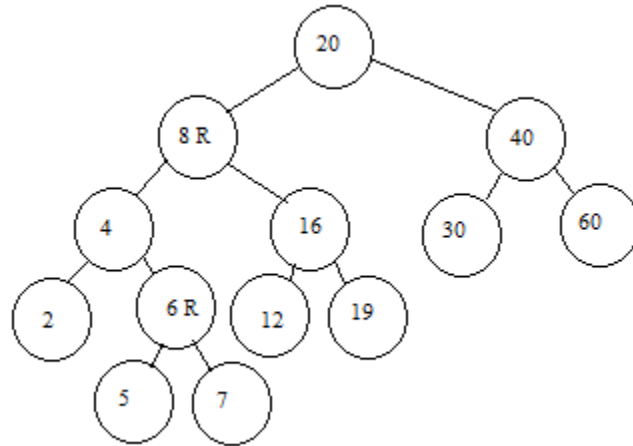
3) (15 pts) Determine the fewest number of multiplications to calculate the product ABCDE, for matrices A, B, C, D, E with the following dimensions:

Matrix	Dimensions
A	2x5
B	5x1
C	1x4
D	4x6
E	6x3

In order to get full credit you must fill out the chart below appropriately, as shown in class. Please include your calculations below the chart.

	A	B	C	D	E
A	0				
B	X	0			
C	X	X	0		
D	X	X	X	0	
E	X	X	X	X	0

4) (10 pts) Draw the result of deleting the value 40 from the red-black tree shown below. In the drawing below, red nodes are indicated with a letter 'R' next to the number stored in the node. In your solution, please put an 'R' in each node that is red. (Note: use the regular binary tree rules for deleting a value which is stored in a node with 2 children.) If you explain your thinking, you may get partial credit for incorrect solutions.



Questions 7 and 8 of the exam are going to look at analyzing the following game:

You are given n k -sided fair dice, each labeled $1, 2, 3, \dots, k$. You roll all of them. Then, separate out the ones that show k . Take the rest and roll them all again. Then, separate out the ones of these that show k , and roll the rest again. Repeat this process until you've separated all the dice out (ie, they all show k). The question we want to analyze is the number of times we expect to roll before completing the game.

7) (14 pts) One way to analyze this is to write a simulation, run it many times and take the average. The latter portion of this is fairly trivial, so for this question, you will simply write a single Java method that takes in n , the number of dice, and k , the number of sides on each dice, simulates this process, and returns the number of turns it took to complete a single simulation of the game. Fill in the method signature given below. Assume that you have access to a static class variable r , that is of type `Random`, which you can use to generate random integers.

```
public static Random r;
```

```
public static int numTurnsSim(int n, int k) {
```

```
}
```

8) (10 pts) Let the dice have k sides each and let $T(n)$ equal the expected number of turns to complete the simulation described previous. A recurrence relation that $T(n)$ satisfies is as follows:

$$T(0) = 0$$

$$T(n) = 1 + \sum_{i=0}^n \left[\binom{n}{i} \left(\frac{1}{k}\right)^i \left(\frac{k-1}{k}\right)^{n-i} T(n-i) \right]$$

In words, explain why this formula is correct. In your explanation, please explain what 1 represents, what the summation index i , represents and what $T(n-i)$ represents. (Of course, explain what each part represents and why their interaction is the way that it is.) Note: One issue with this formula is that a $T(n)$ term appears on the right hand side. But, if one were to solve this recurrence, we can easily take care of this problem by subtracting that term to the left hand side and factoring out $T(n)$. Then, we would have a factor by which we could divide both sides of the resulting equation.

9) (1 pt) Who wrote The Autobiography of Mark Twain? _____

Scratch Page - Please clearly mark what you would like graded on this page.