Spring 2022 COP 3502 Quiz #3 Version A (Algorithm Analysis, Sorting)

Last Name: _____, First Name: _____

Circle Your Recitation: F12:30 F1:30 F2:30 F3:30 F4:30

1) (5 pts) An algorithm processes n values in $O(3^n)$ time. When run on an input of size n = 15, the algorithm takes 100 ms. How long, **in seconds**, is the algorithm expected to take when run on an input of size n = 20?

2) (8 pts) Which of the two following expressions is larger, assuming that n is an integer greater than 1? Provide proof of your answer.

(a) $\sum_{i=1}^{n} i2^i$ (b) $(\sum_{i=1}^{n} i)(\sum_{i=1}^{n} 2^i)$

3) (10 pts) Use the iteration technique to solve the following recurrence relation **EXACTLY.**

$$T(n) = 3T(n-1) + 2, for\ all\ integers\ n > 1$$
$$T(1) = 2$$

Please provide your answer as an exact function that is a closed form representation of T(n).

4) (5 pts) Show the contents of the following array after each iteration of Insertion Sort:

Initial Values	7	1	2	6	5	4	3
1st iteration							
2nd iteration							
3rd iteration							
4th iteration							
5th iteration							
Last iteration	1	2	3	4	5	6	7

5) (4 pts) Show the result of partitioning the array below, using the leftmost element as the partition element. Please use the in-place partitioning algorithm shown in class.

Initial Values	12	19	16	1	3	22	47	14	4	8	31
After Partition											

6) (3 pts) What is the reason that Quick Sort and Merge Sort can both be sped up if we change the base case to be a subarray of size 40 and instead run Insertion Sort on this subarray?

Summation Formulas

$\sum_{i=1}^{n} c = cn \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$, $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}, |x| < 1$

$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$

Scratch Area - Please clearly label any work on this page you would like graded.