

CIS 3362 Homework #5
Number Theory, RSA
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Please work in pairs and put both people's names on each file submitted!

1) Write a program that prompts the user to enter an integer, n , in between 1 and 10^{12} and calculates $\phi(n)$.

2) Using your program from question 1, write a program that determines if (a) an input value in between 1 and 10^{12} is prime, and (b) if so, asks the user to enter an integer in between 1 and the prime number minus 1 and determines if that value is a primitive root. Your program should work as follows:

Calculate each unique prime factor q_i of $p - 1$, and calculate $x^{(p-1)/q_i} \bmod p$ for each q_i . If none of these are equal to 1, then x is a primitive root.

3) A primitive root, α , of a prime, p , is a value such that when you calculate the remainders of α , α^2 , α^3 , α^4 , ..., α^{p-1} , when divided by p , each number from the set $\{1, 2, 3, \dots, p-1\}$ shows up exactly once. Prove that a prime p has exactly $\phi(p-1)$ primitive roots. In writing your proof, you may assume that at least one primitive root of p exists. (Normally, this is the first part of the proof.) (Note: This question is difficult, so don't feel bad if you can't figure it out.)

4) Alice and Bob are using Diffie-Hellman to exchange a secret key. They are using the prime number $p = 1234577$ and the generator $g = 1225529$. Alice picks a secret value a and sends $g^a = 654127$ to Bob. Bob picks a secret value b and sends $g^b = 221505$ to Alice. What is the secret key they share?

5) Decrypt the following message:

20429835450828679741350
26022799626812591980567
30572114224921561344399
14180424833673414562055
19539282983393676142312

These 5 blocks of cipher text were created with a set of RSA public keys that follow:

$n = 43767782750765499923141$
 $e = 986321785648512635467$

When you decrypt, you'll initially get numbers, but those numbers can be converted into blocks of 16 letters each.