

Fall 2021 CIS 3362 Homework #5: Number Theory
Check WebCourses for the due date

- 1) Without the aid of a computer program, determine the prime factorization of 808995600. Show your work. You may do division in a calculator.
- 2) What is $\varphi(808995600)$?
- 3) Using Fermat's Theorem, determine the remainder when 12^{11764} is divided by 1471.
- 4) Using Euler's Theorem, determine $99^{29403} \bmod 34643$.
- 5) Trace through the Miller-Rabin algorithm testing $n = 169$ for primality using the test value of $a = 2$. In particular, first state the value $X = 2^d \pmod{169}$, where d is odd that gets calculated in the algorithm. From that point, the algorithm continually squares X , and depending on the new result, either returns "Probably Prime" or "Composite". Show each value of X in the algorithm and when the decision is made what to return and what is returned.
- 6) Use Fermat Factoring to factor 37001. Fill in the table below (it's possible that more rows than necessary are included.) and the provide the factorization.

X	$X^2 - 37001$	Perfect Square?

7) Define $f(n) = \min\left(\frac{\phi(k)}{k}\right)$, for all positive integers $2 \leq k \leq n$. For example, $f(32) = \frac{4}{15}$, because for each integer in between 2 and 32, the smallest possible value of $\frac{\phi(k)}{k}$ is $\frac{\phi(30)}{30} = \frac{8}{30} = \frac{4}{15}$. Write a program that asks the user for the input value of n , and then prints out $f(n)$ as a fraction in lowest terms. You may assume that the user won't enter a number bigger than 3×10^8 . (Note: You can solve this problem in different ways. A brute force solution will be accepted, but there is a much more elegant solution that runs really fast.) **Please write your program in C, Java or Python and attach it separately.**

Sample Program Run

Please enter n.

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The smallest possible ratio of $\phi(n)/n$ is $4/15$.