

**Fall 2021 CIS 3362 Homework #5: Number Theory**  
**Check WebCourses for the due date**

- 1) Without the aid of a computer program, determine the prime factorization of 808995600. Show your work. You may do division in a calculator.
- 2) What is  $\varphi(808995600)$ ?
- 3) Using Fermat's Theorem, determine the remainder when  $12^{11764}$  is divided by 1471.
- 4) Using Euler's Theorem, determine  $99^{29403} \bmod 34643$ .
- 5) Trace through the Miller-Rabin algorithm testing  $n = 169$  for primality using the test value of  $a = 2$ . In particular, first state the value  $X = 2^d \pmod{169}$ , where  $d$  is odd that gets calculated in the algorithm. From that point, the algorithm continually squares  $X$ , and depending on the new result, either returns "Probably Prime" or "Composite". Show each value of  $X$  in the algorithm and when the decision is made what to return and what is returned.
- 6) Use Fermat Factoring to factor 37001. Fill in the table below (it's possible that more rows than necessary are included.) and then provide the factorization.

X	$X^2 - 37001$	Perfect Square?

7) Define  $f(n) = \min(\frac{\phi(k)}{k})$ , for all positive integers  $2 \leq k \leq n$ . For example,  $f(32) = \frac{4}{15}$ , because for each integer in between 2 and 32, the smallest possible value of  $\frac{\phi(k)}{k}$  is  $\frac{\phi(30)}{30} = \frac{8}{30} = \frac{4}{15}$ . Write a program that asks the user for the input value of n, and then prints out f(n) as a fraction in lowest terms. You may assume that the user won't enter a number bigger than  $3 \times 10^8$ . (Note: You can solve this problem in different ways. A brute force solution will be accepted, but there is a much more elegant solution that runs really fast.) **Please write your program in C, Java or Python and attach it separately.**

**Sample Program Run**

Please enter n.

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The smallest possible ratio of  $\phi(n)/n$  is 4/15.