Geometry

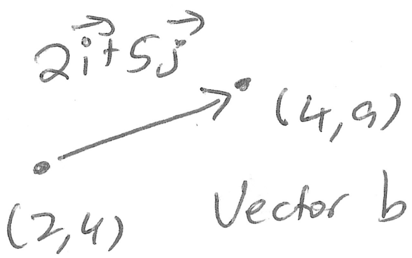
✓1) Vectors
✓2) Dot Product
✓3) Cross Product
✓4) Line, Line Segment Eqn
✓5) Line/Line Intersection (or Line/Seg or Seg/Seg)
✓6) Pt Line /Seg Distance
✓7) Circle-Circle Intersection
✓8) Circle-Line Intersection
✓9) Polygon Area
10) Convex Hull

Vector

Relative Direction of Movement (not absolute pt on Cartesian Plane)

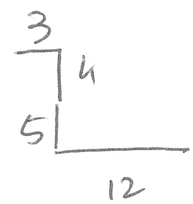
$\vec{i}$ unit vector $+x$

$\vec{j}$ unit vector $+y$

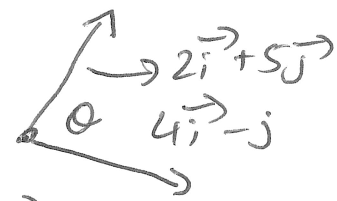


Vector btw to pts

vector from pt1 to pt2 (Direction of movement) $\langle pt2.x - pt1.x,\ pt2.y - pt1.y \rangle$



Dot Product



$V_1 = 2\vec{i} + 5\vec{j}$

$V_2 = 4i - j$

$V_1 \cdot V_2 = 8 - 5$

$V_1 = a\vec{i} + b\vec{j}$

$V_2 = c\vec{i} + d\vec{j}$

$V_1 \cdot V_2 = a \times c + b \times d$ (scalar)

$|V_1| = \sqrt{a^2+b^2}$

$V_1 \cdot V_2 = |V_1||V_2|\cos\theta$

$\cos\theta = \dfrac{V_1 \cdot V_2}{|V_1||V_2|} = \dfrac{3}{\sqrt{29}\sqrt{17}}$ (acos function θ)

* Allows us to calculate angle btw to vectors

Cross Product

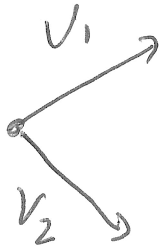
$V_1 = 2\vec{i} + 5\vec{j}$

$V_2 = 4i - \vec{j}$

$2(-1) - 4(5)$

$-2 - 20$

$\boxed{-22}$



$V_1 \times V_2$ returns a vector perpendicular to both v_1 and v_2 with a magnitude equal to the area ~~is~~ that the parallelogram defined by the vectors has.

$\vec{k}$ is unit vector z direction

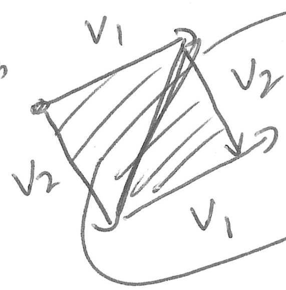
$V_1 = a\vec{i} + b\vec{j}$

$V_2 = c\vec{i} + d\vec{j}$

$V_1 \times V_2 = (ad - bc)\vec{k}$

$|V_1 \times V_2| = ad - bc$

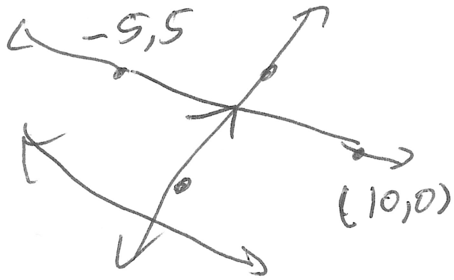
Useful to get parellelogram and triangle areas. $\Rightarrow$ polygon area



has area 22

triangle area $= \dfrac{22}{2} = 11$

Line Eqn



(2,4) , (4,9)

p = # pt on line
d = direction vector of line

$r_1 = (2i + 4j) + \lambda(2i + 5j)$ $\quad r = p + \lambda d$

$= (2 + 2\lambda)i + (4 + 5\lambda)j$

$$\left.\boxed{\begin{aligned} x &= 2 + 2\lambda \\ y &= 4 + 5\lambda \end{aligned}}\right] \text{Parametric Eqn}$$

$r_2 = (-5i + 5j) + \beta(15i - 5j)$

$$\boxed{\begin{aligned} x &= -5 + 15\beta \\ y &= 5 - 5\beta \end{aligned}}$$

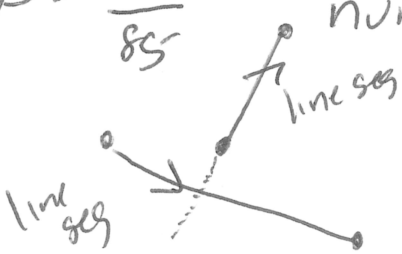
$$\left.\begin{aligned} 2 + 2\lambda &= -5 + 15\beta \\ 4 + 5\lambda &= 5 - 5\beta \end{aligned}\right\} \quad \begin{array}{cccc} a & b & & c \\ 2\lambda & -15\beta & = & -7 \\ 5\lambda & +5\beta & = & 1 \\ d & e & & f \end{array}$$

Kramer's Rule $\quad \text{den} = \begin{vmatrix} 2 & -15 \\ 5 & 5 \end{vmatrix} = 2 \cdot 5 - (5)(-15) = 85$

$\begin{vmatrix} a & b \\ d & e \end{vmatrix}$

$\lambda = \dfrac{-20}{85} = \dfrac{-4}{17}$ $\quad \text{num}\,\lambda = \begin{vmatrix} c & b \\ f & e \end{vmatrix} = \begin{vmatrix} -7 & -15 \\ 1 & 5 \end{vmatrix} = -35 - (-15) = -20$

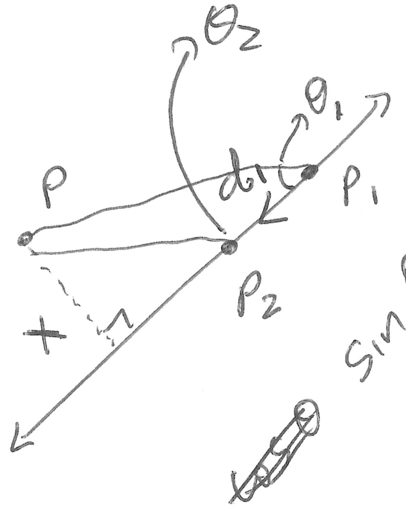
$\beta = \dfrac{37}{85}$ $\quad \text{num}\,\beta = \begin{vmatrix} a & c \\ d & f \end{vmatrix} = \begin{vmatrix} 2 & -7 \\ 5 & 1 \end{vmatrix} = 2 - (-35) = 37$



$x = 2 + 2\left(-\frac{4}{17}\right) = \frac{26}{17}$, $y = 4 + 5\left(-\frac{4}{17}\right) = \frac{48}{17}$

LINE SEG: $0 \le \lambda, \beta \le 1$ $(-EPS, 1 + EPS)$

Pt to Line / Line Seg Distance



$\sin\theta = \dfrac{x}{|\overrightarrow{PP_1}|}$

$x = |\overrightarrow{P_1P}| \sin\theta$

from fact area $\Delta = \frac{1}{2} bc \sin A$

$$\frac{|\underline{d_1} \times (\underline{\overrightarrow{P_1P}})|}{|d_1|} = \boxed{\frac{|d_1||\overrightarrow{P_1P}| \sin\theta_1}{|d_1|} = x}$$

$$x = \frac{|d_1 \times (\overrightarrow{P_1P})|}{|d_1|}$$ pt to line distance

Line Seg Solve

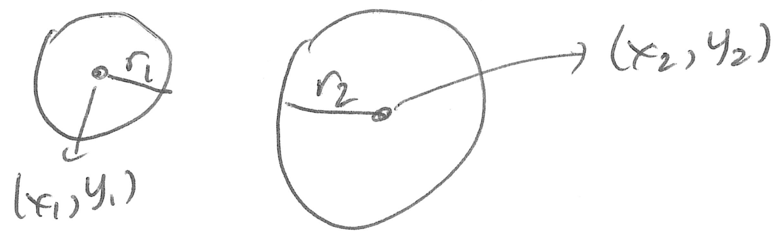
if $\perp$ is on seg, line ans is corect!

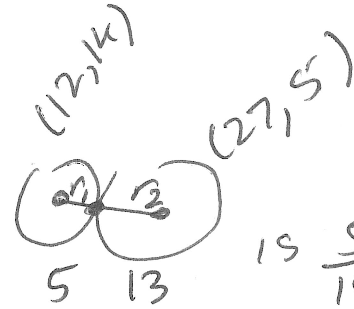
if $\perp$ is NOT seg, ans $\min(|\overrightarrow{PP_1}|, |\overrightarrow{PP_2}|)$.

How to tell: generate line from p with $\perp$ direction. find intersection + see if line's λ, $0 \le \lambda \le 1$. (HARD)

EASY: if $\theta_1 \le 90°$ and $\theta_2 \le 90°$ good.

Circle - Circle Intersection



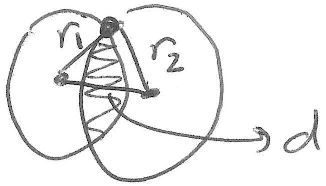


is $\frac{5}{18}$ of the way

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 > (r_1 + r_2)^2$$ NO INTER

$$r = (12i + 14j) + \lambda(15i - 9j)$$

$$(12i + 14j) + \frac{5}{18}(15i - 9j) = \left(\left(12 + \frac{25}{6}\right)\vec{i} + \left(14 - \frac{5}{2}\right)j\right)$$



law of cosines solve for all 3 angles

$$c^2 = a^2 + b^2 - 2ab\cos C$$

to get absolute angle use atan2 $[-\pi, \pi)$

get this area,
sector area - tri area

$$\frac{\theta r^2}{2} - \frac{1}{2}r^2\sin\theta$$

Circle Line Intersection

$$(x-x_1)^2+(y-y_1)^2=r^2$$

$$r=(x',y')+\lambda(\partial x,\partial y)$$

$$x=x'+\lambda\partial x$$
$$y=y'+\lambda\partial y$$

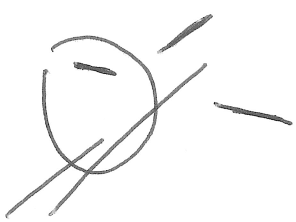
$$(x'+\lambda\partial x-x_1)^2+(y'+\lambda\partial y-y_1)^2=r^2$$

→ # VARS? $1 \rightarrow \lambda$

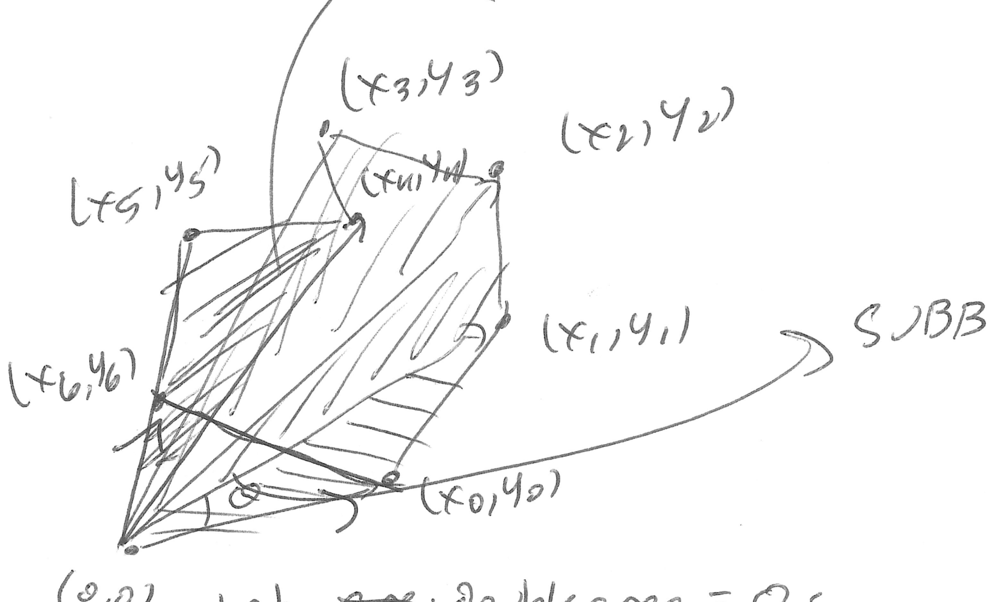
QUADRATIC ~~Es~~ in λ

→ DISCRIMINANT $= b^2-4ac$

$b^2-4ac \geq 0$ before we do formula!



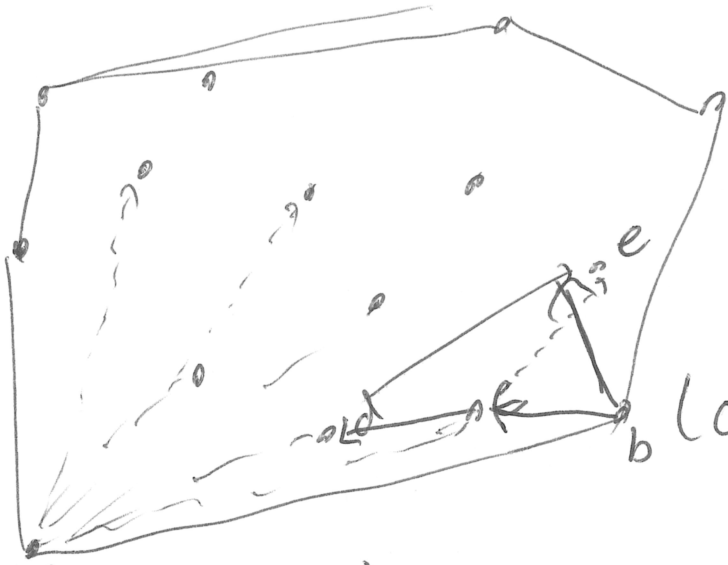
Polygon Area




```
int ~~are~~ doublearea = 0;
for (i = 0; i < n; i++) {
    doublearea += ( x[i] * y[(i+1) % n] -
                    x[(i+1) % n] * y[i] );
}
return abs(doublearea)
```

Convex Hull

Graham Scan



(cross product mag sign)

(bottommost pt)
min Y

1. (bottommost pt) min Y
2. Sort all other pts by angle (atan2)
3. Push bottom pt on stack, Push 1st sorted pt
4. loop through rest of pts.
   while last turn is a left turn
   pop (stack)
   push (item)

b
a

~~d~~	
~~c~~	e
b	b
a	a
stack	S