

ODOMETER PROBLEM

00000

00001

00002

..

00009

00010

....

09999

10000

...

99999

$f(2\ 3\ 5\ _\ _, 3) \rightarrow$ will print out all odometer settings where the first 3 digits are set to 2 3 5.

23500

23501

...

23599

2350_ → do this first f(2 3 5 0 _, 4)
2351_ → do this next f(2 3 5 1 _, 4)
2352_ → do this third f(2 3 5 2 _, 4)
...
2359_ → do this last f(2 3 5 9 _, 4)

printodom(odom, k):

```
    if k == len(odom):  
        print(odom)  
        return
```

```
for i in range(10):  
    odom[k] = i  
    printodom(odom, k+1)
```

Imagine odometer with only 0s and 1s:

Odometer	Set
<u>012</u>	
000	{ }
001	{2}
010	{1}
011	{1,2}
100	{0}
101	{0,2}
110	{0,1}
111	{0,1,2}

Binary Representation of a number

000	0	{ }
001	1	{0}
010	2	{1}
011	3	{1,0}
100	4	{2}
101	5	{2,0}

110	6	{2,1}
111	7	{2,1,0}

$$1011001 = 1x2^6 + 0x2^5 + 1x2^4 + 1x2^3 + 0x2^2 + 0x2^1 + 1x2^0$$

Int x; // x = 5, how do I access the bits of x???

$x \& y \rightarrow$ bitwise and (set intersection)

x = 1010 (10)

y = 1100 (12)

1000 (8)

$x | y \rightarrow$ bitwise or (set union)

x = 1010 (10)

y = 1100 (12)

1110 (14)

The value of n 1s: 1111 is simply $2^n - 1$.

$X \wedge y \rightarrow$ bitwise xor (light switches, grading TF exams)

$X = 1010 \quad (10)$

$Y = 1100 \quad (12)$

$0110 \quad (6)$, 1 in xor means those bits are
Different in x and y,

If $x \wedge y = z$, then $x \wedge z = y$ and $y \wedge z = x$.

How do I look at a single bit and determine if it's on or not?

$a \ll b$, this is left shift a by b bits.

$1010 \ll 3 \rightarrow 1010000$

This is basically multiplying a by 2^b so long as there is no overflow.

$1 \ll n \rightarrow$ this equals 2^n

$a \gg b \rightarrow$ this right shifts a by b bits, chopping off the last b bits

$101011011101 \gg 5 \rightarrow 1010110$

To look at bit i , do this:

if $(x \& (1 \ll i)) \neq 0$:

 // Bit i is on.

Else:

 .. Bit i is off.

$X = 1011100001, i = 4$

$(1 \ll 4) = 0000010000$

A common problem is given n items, say
 $0, 1, 2, \dots, n-1$

List all orderings of those items.

$0, 1, 2$

$0, 2, 1$

$1, 0, 2$

1,2,0

2,0,1

2,1,0

Only difference between this and the odometer is no repeats...

We need to edit the odometer but build in a system to prevent repeats...

SKETCH OF ODOM CODE:

```
If k == length:  
    // PROCESS  
    Return
```

```
For i in range(choices):
```

```
    IF i has already been placed, skip it!
```

```
    Odom[k] = i
```

```
    Odometer(odom, k+1)
```

0   → need some way to skip filling in 0 here

0 1  → here skip BOTH 0 and 1.

Idea → add an used array, where $\text{used}[i] = \text{true}$ if i has already placed in the array, and false if it hasn't.

DERANGEMENTS

0	1	2
1	2	0
2	0	1

For all i ,

$\text{Perm}[i] \neq i$...no person gets their own hat.