

ODOMETER PROBLEM

00000

00001

00002

..

00009

00010

....

09999

10000

...

99999

$f(2\ 3\ 5\ _\ _,\ 3)$ → will print out all odometer settings where the first 3 digits are set to 2 3 5.

23500

23501

...

23599

2350_ → do this first $f(2\ 3\ 5\ 0\ _,\ 4)$

2351_ → do this next $f(2\ 3\ 5\ 1\ _,\ 4)$

2352_ → do this third $f(2\ 3\ 5\ 2\ _,\ 4)$

...

2359_ → do this last $f(2\ 3\ 5\ 9\ _,\ 4)$

`printodom(odom, k):`

```
if k == len(odom):  
    print(odom)  
    return
```

```
for i in range(10):
```

```
    odom[k] = i
```

```
    printodom(odom, k+1)
```

Imagine odometer with only 0s and 1s:

Odometer Set

012

000		{ }
001		{2}
010		{1}
011		{1,2}
100		{0}
101		{0,2}
110		{0,1}
111		{0,1,2}

Binary Representation of a number

000	0	{ }
001	1	{0}
010	2	{1}
011	3	{1,0}
100	4	{2}
101	5	{2,0}

110	6	{2,1}
111	7	{2,1,0}

$$1011001 = 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

Int x; // x = 5, how do I access the bits of x???

$x \& y \rightarrow$ bitwise and (set intersection)

$$\begin{array}{rcl}
 x = 1010 & (10) \\
 y = 1100 & (12) \\
 \hline
 1000 & (8)
 \end{array}$$

$x | y \rightarrow$ bitwise or (set union)

$$\begin{array}{rcl}
 x = 1010 & (10) \\
 y = 1100 & (12) \\
 \hline
 1110 & (14)
 \end{array}$$

The value of n 1s: 1111 is simply $2^n - 1$.

$X^y \rightarrow$ bitwise xor (light switches, grading TF exams)

$X = 1010 \quad (10)$

$Y = 1100 \quad (12)$

$0110 \quad (6)$, 1 in xor means those bits are
Different in x and y,

If $x^y = z$, then $x^z = y$ and $y^z = x$.

How do I look at a single bit and determine if it's on or not?

$a \ll b$, this is left shift a by b bits.

$1010 \ll 3 \rightarrow 1010000$

This is basically multiplying a by 2^b so long as there is no overflow.

$1 \ll n \rightarrow$ this equals 2^n

$a >> b \rightarrow$ this right shifts a by b bits, chopping off the last b bits

101011011101 $>> 5 \rightarrow 1010110$

To look at bit i, do this:

```
if (x & (1<<i)) != 0:  
    // Bit i is on.
```

```
Else:  
    .. Bit i is off.
```

$X = 1011100001, l = 4$
 $(1<<4) = 0000010000$

A common problem is given n items, say 0, 1, 2, ..., n-1

List all orderings of those items.

0,1,2

0,2,1

1,0,2

1,2,0

2,0,1

2,1,0

Only difference between this and the odometer is no repeats...

We need to edit the odometer but build in a system to prevent repeats...

SKETCH OF ODOM CODE:

```
If k == length:  
    // PROCESS  
    Return
```

For i in range(choices):

IF i has already been placed, skip it!

Odom[k] = i

Odometer(odom, k+1)

0 → need some way to skip filling in 0 here

0 1 → here skip BOTH 0 and 1.

Idea → add an used array, where $\text{used}[i] = \text{true}$ if i has already placed in the array, and false if it hasn't.

DERANGEMENTS

0 1 2

1 2 0

2 0 1

For all i ,

$\text{Perm}[i] \neq i \dots$ no person gets their own hat.