

# Problem H: Arup and the Magical Ratio Merge

Filename: merge

Time limit: 5 seconds

Arup is investigating the ancient Rituals of Ratio in the Temple of Integers, a mysterious sanctuary where a sequence of numbers hides an evolving pattern of merging. As part of his research, Arup stumbles upon a sacred scroll with a strange instruction:

"Given a subset of numbers from 1 to n, start by placing each number in its own group. Then, proceed in rounds, where each round corresponds to an integer  $k = 2, 3$ , and so on.

In each round:

- Scan all current groups.
- If two groups contain numbers  $a$  and  $b$  such that  $a / b = k$  or  $b / a = k$ , merge those two groups into one (e.g., merging  $\{1, 2\}$  and  $\{3\}$  gives  $\{1, 2, 3\}$ ).
- Continue merging within the same round until no more valid merges can be made for that value of  $k$ .

Once no further merges are possible in a round, increment  $k$  and repeat the process for the next round."

Arup becomes curious. He wants to understand how long this magical merging process will take and how chaotic it becomes. Specifically, for a given subset of numbers:

- How many rounds will be executed until no more merges are possible?
- What is the cumulative sum of the number of disjoint sets across all rounds, **NOT** including the initial state?

To answer these questions and impress the temple sages, Arup turns to you for help.

## The Problem

Given a subset of positive integers, simulate the merging process as described above. At the beginning, each number starts in its own group. In each round  $k = 2, 3, \dots$ , you can merge any two groups containing numbers  $a$  and  $b$  if  $a / b = k$  or  $b / a = k$ . Multiple merges can happen in the same round, but each merge combines exactly two distinct groups. The round ends when no more merges can be performed for that round's  $k$ .

Determine:

- The total number of rounds until no merges are possible.
- The cumulative sum of the number of disjoint sets across all rounds (including the initial one).

## The Input

- The first line contains an integer  $c$  — the number of test cases.
- The sum of sizes of all subsets across test cases will not exceed  $10^5$ .

Each test case is structured as follows:

- The first line contains an integer  $m$  — the size of the subset.
- The second line contains  $m$  distinct integers  $s_1, s_2, \dots, s_m$  — the elements of the subset.

## The Output

For each test case, output two integers separated by a space:

- The number of rounds until no more merges are possible.
- The cumulative sum of the number of disjoint sets across all rounds.

## Input Bounds and Corresponding Credit

30 Points	70 Points
<ul style="list-style-type: none"><li>• <math>1 \leq c \leq 100</math></li><li>• <math>1 \leq m \leq 10</math></li><li>• <math>1 \leq s_i \leq 1000</math></li><li>• All <math>s_i</math> are distinct within a test case.</li></ul>	<ul style="list-style-type: none"><li>• <math>1 \leq c \leq 110</math></li><li>• <math>1 \leq m \leq 100</math></li><li>• <math>1 \leq s_i \leq 10000</math></li><li>• All <math>s_i</math> are distinct within a test case.</li></ul>

## Samples

Input	Output
2	2 3
5	0 0
1 2 3 4 6	
4	
2 3 5 7	

**Sample Explanation:** In the first sample, we first merge  $\{1, 2\}$ ,  $\{3, 6\}$  and also merge  $\{4\}$  with  $\{1, 2\}$ . Thus, when we complete the round with  $k = 2$ , our sets are  $\{1, 2, 4\}$  and  $\{3, 6\}$ . At this stage, there are 2 sets. When  $k = 3$ , we merge these two sets because  $6 = 3 \times 2$ , so when this round completes, we have one set  $\{1, 2, 3, 4, 6\}$ . At this point, we've completed 2 rounds and no more merges are possible, thus the answer to the first question is 2, and the answer to the second question is  $2 + 1 = 3$ , the sum of the number of sets at the end of the two rounds described.

In the second sample, no merging ever occurs, because no value in the original set is a multiple of another value in the original set.