Foundation Exam Practice Questions (Dec 2011 exam) - Solutions + Grading Criteria

1) (10 pts) **Recursion.** Write a **recursive** function that deletes every other node in a linked list pointed to by head, which is a parameter to the function. Specifically, make sure you delete the second, fourth, sixth, etc. nodes and return a pointer to the front of the new list. If the list has zero or one item in it, the list should be unchanged and a pointer to its front should be returned. Your function should make use of the following struct node and function prototype:

```
struct node {
      int data;
      struct node *next;
};

struct node* delEveryOther(struct node *head) {

      // Base case
      // Grading:  1 point
      if (head == NULL || head->next == NULL)
            return head;

      // Assign a temp pointer to the node to be deleted
      struct node* temp = head->next;  // Grading:  1 points

      // Bypass the node to be deleted.
      head->next = temp->next;                // Grading:  2 points
      // Can also do:  head->next = head->next->next;

      // Free the temp node
      free(temp);                             // Grading:  1 points

      // Recursively call the function on the rest of the list
      head->next = delEveryOther(head->next); // Grading 4 points
      // Can also do:  delEveryOther(head->next);

      // Return the original front of the list
      return head;                            // Grading 1 point


}
```

2) (15 pts) PRF (Induction)

Prove, using mathematical induction, that for all non-negative integers n, $10 \mid (9^{n+1} + 13^{2n})$.

Base case: n = 0. Plug into the expression to get $9^{0+1} + 13^{2(0)} = 9 + 1 = 10$. Since $10 \mid 10$, the base case holds. (2 pts)

Inductive Hypothesis (IH): Assume for an arbitrary non-negative integer n = k, that $10 \mid (9^{k+1} + 13^{2k})$. Equivalently, there is some integer c such that $10c = 9^{k+1} + 13^{2k}$. (2 pts)

Inductive Step: Prove for n = k+1 that $10 \mid (9^{k+1+1} + 13^{2(k+1)})$. (2 pts)

$9^{k+1+1} + 13^{2(k+1)}$	$= 9^{k+2} + 13^{2k+2}$	**(1 pt)**
	$= 9^1 9^{k+1} + 13^2 13^{2k}$, **since** $a^{b+c} = a^b a^c$.	**(1 pt)**
	$= 9(9^{k+1}) + 169(13^{2k})$	**(1 pt)**
	$= 9(9^{k+1}) + (160 + 9)(13^{2k})$	**(1 pt)**
	$= 9(9^{k+1}) + 9(13^{2k}) + 160(13^{2k})$	**(1 pt)**
	$= 9(9^{k+1} + 13^{2k}) + 160(13^{2k})$	**(1 pt)**
	$= 9(10c) + 160(13^{2k})$, **using the integer c defined in the IH (2 pts)**	
	$= 10(9c + 16(13^{2k}))$	**(1 pt)**

Since 9c, 16 and 13^{2k} are all integers, it follows that the expression above is divisible by 10, proving the inductive hypothesis.

<u>Follow Up Questions to Activity</u>

1) What are the challenges in grading?

2) What can you do as a student to help the grader do their job?

3) What sorts of items are points awarded for?

4) What sorts of syntax issues result in a loss of points and which syntax errors typically don't? Why?

5) What indication/clue do we have that might help us come up with splitting 169 in the induction question into 160 and 9?