Computer Science Foundation Exam

August 13, 2010

Section I A

COMPUTER SCIENCE

**NO books, notes, or calculators may be used,
and you must work entirely on your own.**

Name: _____

PID: _____

Question #	Max Pts	Category	Passing	Score
1	10	DSN	7	
2	10	ANL	7	
3	10	ALG	7	
4	10	ALG	7	
5	10	ALG	7	
TOTAL	50			

You must do all 5 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat.

1) (10 points) **Recursion** Write a **recursive** function that sums the *odd* digits of an integer. For example, if your function takes in the number 8135267, it should return 16 because 1+3+5+7=16. (Note that 8, 2 and 6 were ignored since they are even.) You may assume that the function will only be passed non-negative integers.

```
int sumOddDigits(int number)
{




}
```

2) (10 points) **Summations**

a) Determine a simplified closed-form solution for the following summation in terms of *n*:

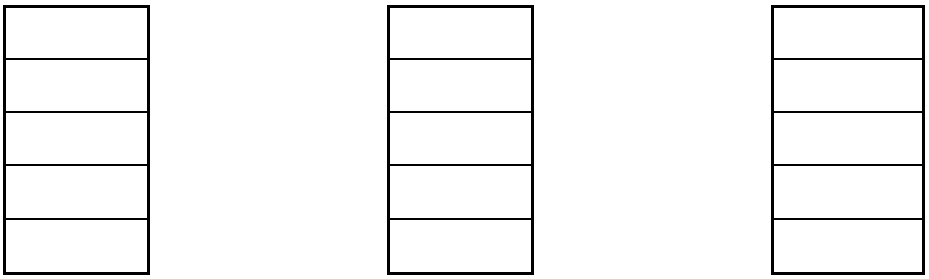
$$\sum_{i=1}^{n}\sum_{j=1}^{n}(5i+3j)$$

b) Evaluate the following summation: $\sum_{i=11}^{20} 3i$

3) (10 points) **Stack Applications** Evaluate the postfix expression below using a stack. Show the contents of the stack at the indicated points 1, 2 and 3 in the postfix expression.

(1) (2) (3)

8 6 8 * 9 5 - / 1 4 + 2 * - /

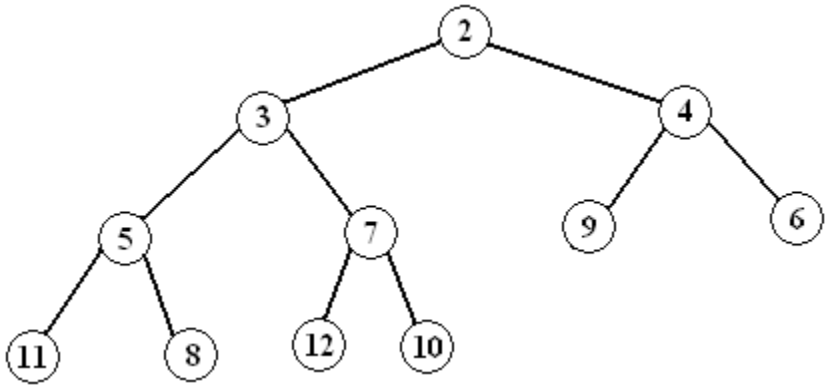


1 2 3

Resulting answer:

_____

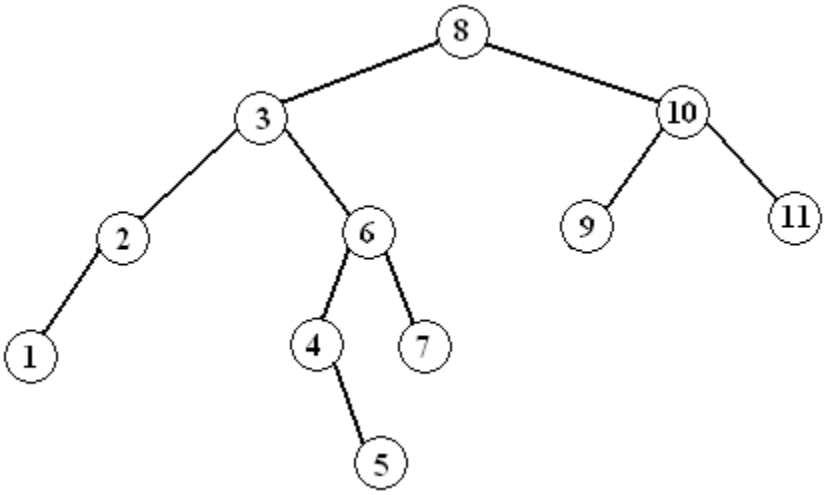
4) (10 points) **Binary Heaps** Consider the following binary min-heap:



a) Show the state of the heap after inserting the value 1. Show both the state before any percolations and the final state.

b) Show the result of deleting the minimum value from the original heap (i.e. ignore part a when answering this part). Show both the state before any percolations and the final state.

5) (10 points) **Binary Tree Traversals**



Give the preorder, inorder, and postorder traversals of the binary tree shown above.

Preorder:

___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___

Inorder:

___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___

Postorder:

___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___ , ___

Why is the tree depicted above not a valid AVL tree?

Computer Science Foundation Exam

August 13, 2010

Section I B

COMPUTER SCIENCE

**NO books, notes, or calculators may be used,
and you must work entirely on your own.**

Name: _____

PID: _____

Question #	Max Pts	Category	Passing	Score
1	10	ANL	7	
2	10	DSN	7	
3	10	DSN	7	
4	10	ALG	7	
5	10	ALG	7	
TOTAL	50			

You must do all 5 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat.

(10 x 1 pt): Analysis

Indicate the time complexity for each of the following operations in terms of Big-O notation, assuming that efficient implementations are used. Give the *worst case* complexities. Following notations are being used:

AINC is an array containing **`n`** integers arranged in increasing order.
AD is an array containing **`n`** integers arranged in decreasing order.
AR is an array containing **`n`** integers in random order.
Q is a queue implemented as a linked list and containing **`p`** elements.
LINK is a linked list containing **`n`** nodes.
CIRC is a circular linked list containing **`n`** elements, where **`C`** points to the last element.
T is a binary search tree containing **`n`** nodes.

a) Searching for an element in AINC using linear search. _____

b) Deleting the 10^{th} node of linked list LINK. _____

c) Calling a function which uses Q, and calls *dequeue* **m** times. _____

d) Inserting an element at the end of the list CIRC. _____

e) Deleting the last element of CIRC. _____

f) Finding the largest element of T. _____

g) Determining the height of T. _____

h) Making the call selectionsort (AINC, n). _____

i) Making two calls one after another. The first call is mergesort(AD,n), followed by the call insertionsort(AD,n). _____

j) Converting a decimal integer *num* into its binary equivalent. _____

2) (10 points) Design

Write a recursive function that will find the height of a binary tree. The height of an empty tree is defined as -1. The height of a single node tree is defined as 0.

```
struct treeNode {
   int data;
   struct treeNode *left, *right:
};


int height (struct treeNode *ptr)  {









}
```

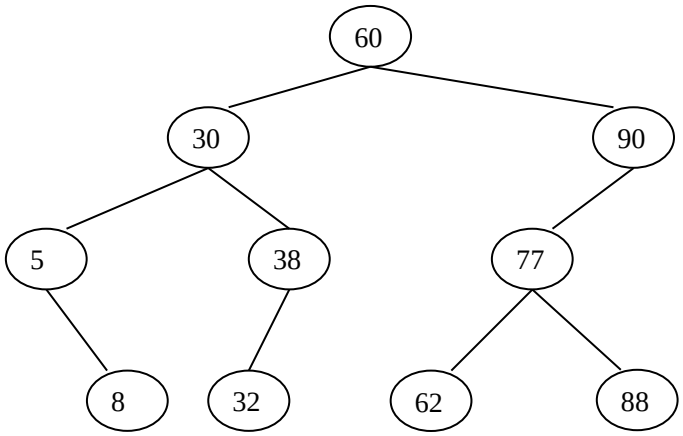
3) (10 points) Design
Write a function which accepts a linear linked list J and converts it into a circular linked list. The function should return a pointer to the last element. The function prototype is provided for you below.

The node structure is as follows:

```
struct listNode {
     int data;
     struct listNode *next;
};

struct listNode* convert(struct listNode* J) {




}
```

4) (10 points) Algorithm

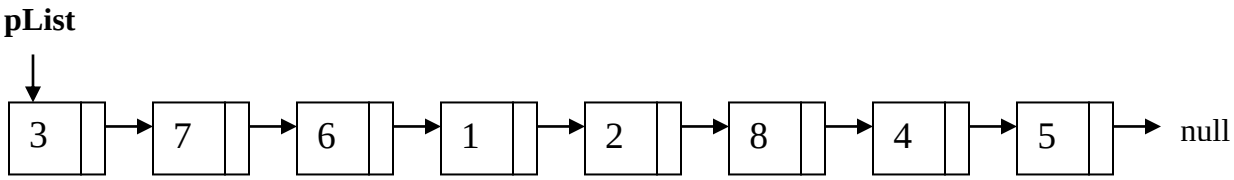
Given the binary tree shown below, determine the order in which the nodes of the binary tree shown above are visited assuming the function **A(root)** is invoked. Assume that the tree nodes and pointers are defined as shown. Assume that **root** is a pointer to the node containing 60. Place your answers in the boxes provided.




```
struct treeNode{
   int data;
   struct treeNode *left, *right:
}
struct treeNode *tree_ptr;

void A(struct treeNode *node_ptr){
    if (node_ptr != NULL){
       printf("%d ,",node_ptr->data);
       B(node_ptr->left);
       B(node_ptr->right);
    }
}

void B(struct treeNode *node_ptr){
    if (node_ptr != NULL) {
       A(node_ptr->left);
       printf("%d ,",node_ptr->data);
       A(node_ptr->right);
   }
}
```

ANSWER:

5) (10 points) Algorithm Consider the linked list shown below where `pList` points to the node containing the value 3. Redraw the list showing the changes to the list after the following code is executed




```
pCur= pList;
while ( pCur->next->next != NULL)
       pCur = pCur->next;
pCur->next->next = pList;
pList = pCur->next;
pCur->next = NULL;
pCur = NULL;
```

Computer Science Foundation Exam

August 13, 2010

Section II A

DISCRETE STRUCTURES

NO books, notes, or calculators may be used,

and you must work entirely on your own.

Name: _____

PID: _____

Question #	Max Pts	Category	Passing	Score
1	15	PRF (Induction)	10	
2	15	PRF (Sets)	10	
3	10	PRF (Logic)	6	
ALL	40	---	26	

You must do all 3 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and <u>not</u> graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all <u>be neat</u>.

1) (15 pts) PRF **(**Induction)

Prove for all positive integers n, $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 3^n & 3^n - 1 \\ 0 & 1 \end{bmatrix}$.

2) (15 pts) PRF **(**Sets)

Disprove the following three assertions about arbitrary chosen sets A, B and C with the use of a counterexample for each one. (You can use different counterexamples for all three parts.)

(a) If $A \subseteq B \cup C$, then either $A \subseteq B$ or $A \subseteq C$.
(b) If $(B - A) \subseteq (C - A)$, then $B \subseteq C$.
(c) If $(A \cup B) \subseteq (A \cap C)$, then $B = \varnothing$.

3) (10 pts) (PRF) Logic

Simplify the following logical expression as much as possible using the laws of logic only. Show each step and state which rule is being used. (Note: You may combine both associative and commutative in a single step, so long as you do so properly.)

$p \vee [p \wedge [\neg(\neg r \vee \neg q) \vee (\neg r \wedge q)]]$

Computer Science Foundation Exam

August 13, 2010

Section II B

DISCRETE STRUCTURES

NO books, notes, or calculators may be used,

and you must work entirely on your own.

Name: _____

PID: _____

Question	Max Pts	Category	Passing	Score
4	15	CTG (Counting)	10	
5	15	PRF (Relations)	10	
6	15	PRF (Functions)	10	
7	15	NTH (Number Theory)	10	
ALL	60	---	40	

You must do all 4 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and <u>not</u> graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all <u>be neat</u>.

4) (CTG) Counting (15 pts)

(a) (5 pts) Round robin play is being set up for a chess tournament. There are 160 participants broken up into ten groups of 16. Within each group, each pair of participants plays each other exactly once. How many total chess games will be played in the round robin portion of this tournament?

(b) (10 pts) Determine the number of non-negative integer solutions to the equation

$$a + b + c + d + e = 70.$$

5) (PRF) Relations (15 pts)

Let R be a relation over the positive integers defined as follows:

R = {(a,b) | gcd(a,b) > 1 but $a \nmid b$ and $b \nmid a$ }

(a) (5 pts) In laymen's terms, describe how to determine whether or not two positive integers are related via R.

(b) (10 pts) Determine whether or not R satisfies the following properties. Give a brief justification for each of your answers.

(i) reflexive
(ii) irreflexive
(iii) symmetric
(iv) anti-symmetric
(v) transitive

6) (PRF) Functions (15 pts)

(a) (7 pts) Explain why the function $f(x) = x^2 - 8x + 12$ with a domain of the real numbers is not an injective function. Determine a restriction on the domain of the form $[a, \infty)$, where a is a real number that would make this function an injection. Attempt to minimize a. Namely, for your answer a, it should be the case that a domain $[a - \varepsilon, \infty)$, where $\varepsilon > 0$ would make the function not injective.

(b) (8 pts) For the injective function described by f(x) in part (a) with the domain you've determined for that function, find $f^{-1}(x)$.

7) (NTH) Number Theory (15 pts)

(a) (5 pts) Two integers are defined as "relatively prime" to one another if they share no common factors. (For example, 12 and 35 are relatively prime but 18 and 33 are not.) Determine how many positive integers less than 24 are relatively prime to 24.

(b) (10 pts) Find at least one integer solution (for x and y) to the equation $117x + 48y = 3$.