Recurrence Relations to Solve

1) $T(n) = 2T\left(\frac{n}{2}\right) + 1, T(1) - 1$

2) $T(n) = T(n-1) + n, T(1) = 1$

3) $T(n) = T\left(\frac{n}{2}\right) + n, T(1) = 1$

4) $T(n) = 2T\left(\frac{n}{2}\right) + n, T(1) = 1$

Solution to #1 using iteration technique

Original equation: $T(n) = 2T\left(\frac{n}{2}\right) + 1$
Plugging in for $\frac{n}{2}$, we get $T\left(\frac{n}{2}\right) = 2T\left(\frac{\frac{n}{2}}{2}\right) + 1 = 2T\left(\frac{n}{4}\right) + 1$
Similarly, we find:

$$\begin{aligned} T(n) &= 2T\left(\frac{n}{2}\right) + 1 \\ &= 2\left(2T\left(\frac{n}{4}\right) + 1\right) + 1 \\ &= 4T\left(\frac{n}{4}\right) + 2 + 1 \\ &= 4T\left(\frac{n}{4}\right) + 3 \end{aligned}$$

Repeat, plugging in $T\left(\frac{n}{4}\right)$:

$$\begin{aligned} &= 4\left(2T\left(\frac{n}{8}\right) + 1\right) + 3 \\ &= 8T\left(\frac{n}{8}\right) + 4 + 3 \\ &= 8T\left(\frac{n}{8}\right) + 7 \end{aligned}$$

In general, after k steps, we get:

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + (2^k - 1)$$

If we let $2^k = n$ (so that $k = log_2 n$), we get

$$T(n) = nT\left(\frac{n}{n}\right) + (n-1) = n(1) + (n-1) = 2n - 1 = O(n)$$

Yielding the Big-Oh bound of the recurrence relation.