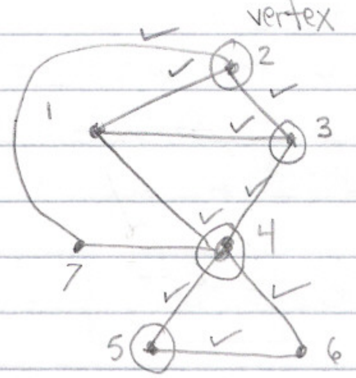
NOTES 04/18/05

3SAT $\leq_p$ VERTEX-COVER

Assuming a solution to Vertex-Cover we will solve 3-SAT in poly-time, given an input w to 3-SAT, we will construct a function (in poly-time) f such that $f(w) \in$ V-C iff $w \in$ 3-SAT.

Vertex-Cover: Given a graph G and an integer k, does there exist a vertex cover of G using k vertices?



2, 3, 4, 5

There exists a Vertex-Cover using 4 vertices

Create a Graph from a 3-SAT

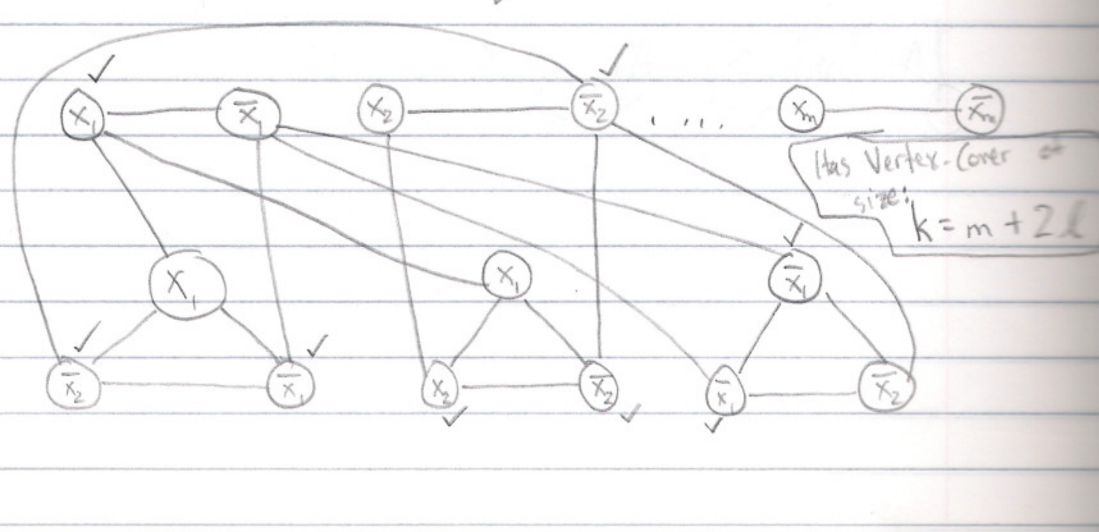
3-SAT: $(x_1 \vee x_2 \vee \bar{x}_3) \wedge (\quad)$

boolean variables: $x_1, x_2, x_3, \ldots x_n$

\# clauses = ℓ

\# variables = m

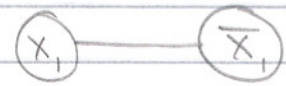
must pick one vertice in every bar



must pick two of three vertices in triangle

To generate graph:

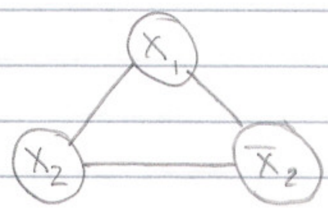
① Create "bars" for every variable



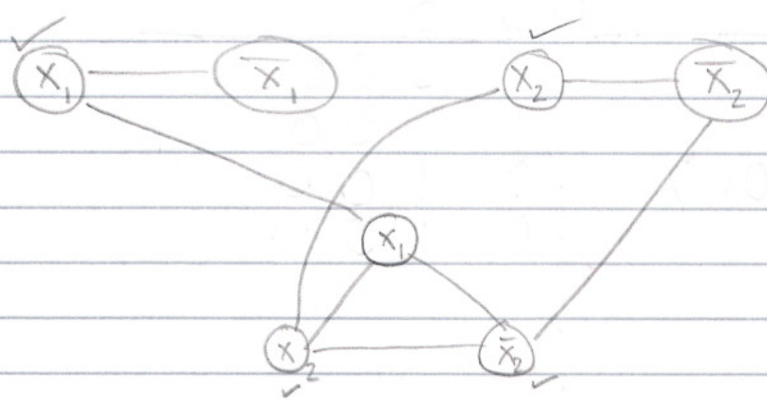
② Draw a triangle formation of each clause:

$$(x_1 \vee x_2 \vee \bar{x}_2)$$

$$\Downarrow$$



③ Connect the corresponding nodes.
Select one node from each bar, and two nodes from each triangle.



$3SAT \leq_p$ SUBSET-SUM

$f(w)$ → boolean formula

$f(w)$ → instance of subset sum

Variables: $x_1, x_2, \dots x_l$

Clauses: $c_1, c_2, \dots c_k$

$$(x_1 \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee x_3 \vee \overline{x_1}) \wedge \dots$$

Integer Representation | Clauses

For each clause column mark a 1 for each variable that appears in it

Variables		1	2	3	l	c_1	c_2	....	c_k
x_1	y_1	1	0	0	0	1	0	....	
$\overline{x_1}$	z_1	1	0	0	0	0	1		
x_2	y_2		1	0 ...	0	0	1		
$\overline{x_2}$	z_2		1	0 ...	0	1	0		
x_3	y_3					1	1		
$\overline{x_3}$	z_3					0	0		
⋮						⋮	⋮		
	y_l				1	0	0		
	z_l				1	0	0		
	g_1	0	0			1	0	0	0
	h_1					1	0	0	0
	g_2	/* all 0's */				0	1	0	0
	h_2					0	1	0	0
							⋮	. . . . .	

C_1 C_2

$$(x_1 \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee x_3 \vee \overline{x_1})$$

	256 (4^4) 1	64 (4^3) 2	16 (4^2) 3	(4^1) c_1	1 (4^0) c_2	
y_1	1	0	0	1	0	260
z_1	1	0	0	0	1	257
y_2	0	1	0	0	1	65
z_2	0	1	0	1	0	68
y_3	0	0	1	1	1	21
z_3	0	0	1	0	0	16
g_1	0	0	0	1	0	4
h_1	0	0	0	1	0	4
g_2	0	0	0	0	1	1
h_2	0	0	0	0	1	1
	1	1	1	3	3	

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