LA Session - Final Exam Review (Fall 2018 Exam)

1) For how many combinations of variable settings for p, q and r does the expression $\bar{p} \wedge (\overline{q \vee r})$ evaluate to true?

a) 0 b) 1 c) 2 d) 3 e) None of the above

2) Which of the following Boolean expressions is equivalent to $(\overline{p \wedge q}) \wedge ((\bar{p} \vee r) \vee \bar{q})$?

a) $p \vee q$ b) $p \wedge q$ c) $\bar{p} \wedge \bar{q}$ d) r e) None of the above

3) Consider the following open statement: $P(x, y) = \forall x[\exists y | xy = 0]$. Assuming that the universe of values that x and y can be selected from are the same, for which of the following sets is the open statement true?

a) Z^+ b) Z^- c) {1,2,3} d) Z e) Q^+

4) Let A = {1, 2, 3, 4} and B = {2, 8, 9}. What is the value of $|\wp(A \cup B)|$?

a) 6 b) 7 c) 64 d) 128 e) None of the above

5) Let A = {1, 2, 3, 4} and B = {2, 8, 9}. Which of the following is an element of A x B?

a) 1 b) (4,8) c) (2,1) d) (8,4) e) None of the above

6) Assume that sets A, B and C are finite and that there exist some elements in the intersection of all three sets and that the number of elements in the intersection of each pair of these sets is strictly greater than the number of elements in the intersection of all three sets. Which of the following quantities is the smallest?

a) $|A - (B - C)|$ b) $|A \cup B \cup C|$ c) $|A - B - C|$

d) $|A \cup (B \cap C)|$ e) $|A \cap (\mathcal{U} - B)|$

7) What is the remainder when 19^{12} is divided by 16?

a) 1 b) 3 c) 9 d) 11 e) None of the above

8) When writing out the steps to the Euclidean Algorithm to find the GCD between 173 and 47, how many equations are written with a positive remainder? (Note: assume that we have 173 by itself on the left hand side for the first equation of the algorithm.)

a) 2 b) 3 c) 4 d) 5 e) None of the above

9) Alice averages 15 miles per hour driving from Orlando to Tampa. She averages 45 miles per hour on her drive back from Tampa to Orlando. What was her average speed, in miles per hour, for the round trip?

a) 20 b) 25 c) 30 d) 40 e) None of the above

10) Which of the following is equal to $\sum_{i=0}^{n}(2i+1)$, assuming that n is a positive integer?

a) n^2 b) n^2+2n+1 c) $2n^2-n$ d) $2n^2+n$ e) None of the above

11) Let matrix A have 3 rows and 7 columns. Let matrix B have 7 rows and 5 columns. What are the dimensions of the product A x B? (Dimensions are listed as ordered pairs with rows followed by columns.)

a) (3, 5) b) (3, 7) c) (5, 3) d) (7, 7) e) None of the above

12) Which of the following integrals provides an upper bound to the sum $\sum_{i=1}^{n}\ln(i)$?

a) $\int_1^n \frac{1}{x}dx$ b) $\int_1^n lnxdx$ c) $\int_1^n \ln(x+1)dx$ d) $\int_{.5}^{n-.5} lnxdx$ e) None of the above

13) A bag contains 5 green Skittles, 3 red Skittles and 7 yellow Skittles. In how many different ways can you choose 3 Skittles, one of each color?

a) 3 b) 5 c) 7 d) 15 e) 105

14) John gets a 100% on his geography quizzes 80% of the time. In the third nine weeks he took 5 geography quizzes, which of the following is the probability that he got a 100% on three of them?

a) $(.8)^3$ b) $5(.8)^3(.2)^2$ c) $10(.8)^3(.2)^2$ d) $(.8)^5$ e) None of the above

15) How many permutations of the letters in ACEVEDO contain consecutive vowels?

a) $\frac{7!}{2}-3(4!)$ b) $7!-3(4!)$ c) 72 d) $4(6!)$ e) None of the above

16) A chicken store sells four types of items: nuggets, strips, filets and legs. You've decided that you want to buy exactly 12 items and that you want at least 4 strips. Assume that the store has plenty of each type of item in stock. How many different orders can you make? (Two orders are different if they have a different quantity of at least one type of item. One valid order for this query is 3 nuggets, 5 strips, 2 filets and 2 legs.)

a) $\binom{12}{4}$ b) $\binom{12}{8} - \binom{8}{4}$ c) $\binom{11}{4}$ d) $\binom{8}{4}$ e) None of the above

17) John is late to school 5% of the time. Given that he is late, he receives a detention 20% of the time. Given that he is on time, he still receives a detention 10% of the time. John has received a detention on a particular day. What is the probability that he was late on that day?

a) $\frac{1}{10}$ b) $\frac{1}{19}$ c) $\frac{2}{19}$ d) $\frac{2}{21}$ e) None of the above

18) Trisha rolls two dice – one is a regular six-sided die with labels 1 through 6, and the other is a four-sided die with labels 1 through 4. Both dice are fair, so each of the possible labels on a single die is equally likely to show on any given roll. Trisha rolls both dice simultaneously. What is the probability that the sum of the labels showing is less than 7?

a) $\frac{7}{8}$ b) $\frac{7}{12}$ c) $\frac{3}{4}$ d) $\frac{7}{24}$ e) None of the above

19) Ten years ago, John was twice as old as Mary. In five years, John will only be 50% older than Mary. What is the sum of their ages right now?

a) 45 b) 65 c) 75 d) 85 e) None of the above

20) John can build a house in 15 days. Jim can build a house in 20 days. They hire Jonah and all together, the three of them build a house in 5 days. How many days would it take Jonah to build the house by himself?

a) 10 b) 15 c) 16 d) 18 e) None of the above

21) Let $f(x) = 3x + 4$ and $g(x) = (x - 2)^2$. What is $g(f(x))$?

a) $3(x - 2)^2 + 4$ b) $3x^2 + 12x + 4$ c) $9x^2 + 12x + 4$

d) $3x^2 + 2$ e) None of the above

22) Let R be a relation over A x A where A = {1, 2, 3, 4}.
In particular, let R = {(1, 2), (2, 2), (2, 3), (3, 2), (3, 3), (3, 4), (4, 1), (4, 4)}. What ordered pair must be added to R to make it reflexive?

a) (1, 1) b) (2, 2) c) (1, 4) d) (2, 4) e) None of the above

23) Let A = {1, 2, 3, 4, 5} and B = {a, b, c}. How many surjective functions are there with the domain of A and co-domain of B?

a) 3^5 b) $3^5 - 2^5$ c) $3^5 - 3(2^5)$ d) $3(3^4 - 2^5 + 1)$ e) None of the above

24) What is the name given to a relation that is reflexive, anti-symmetric and transitive?

a) equivalence relation b) partial ordering relation c) quiggy relation

d) symbiotic relation e) None of the above

25) James Madison is known as the "Father of the Constitution." What major document which outlines the basis of the United States government did Madison help write?

a) Constitution b) Declaration of Independence c) Gettysburg Address

d) Missouri Compromise e) Farmer's Almanac

2018 Fall 2018 COT 3100 Section 2 Final Exam - Free Response

Last Name: _____, First Name : _____
Please show your work and put a box around your final answer for each question.

1) (8 pts) Jaylon is 7 miles south of a stream that flows due east. He is also 12 miles west and 21 miles north of his apartment. He wishes to go to the stream to collect some water and then return to his apartment. What's the minimum distance he can travel to accomplish this task? (Hint: the result is an integer number of miles.)

2) (8 pts) Find the ordered pair (x,y) that satisfy the pair of equations shown below:

$$log_2 x^3 + log_4 y^2 = 6$$

$$log_4 x^4 + log_2 y^6 = 20$$

3) (12 pts) Recall that $\frac{d}{dx}\sin(x) = \cos(x)$ and $\frac{d}{dx}\cos(x) = -\sin(x)$. We denote the n^{th} derivative of a function as $\frac{d^n y}{dx^n}$. Let $y = \sin(2x)$. Conjecture a guess for the function $\frac{d^{4n} y}{dx^{4n}}$, using the function y given, for all non-negative integers n. (Thus, your formula will be a formula for the 0^{th}, 4^{th}, 8^{th}, 12^{th}, etc. derivatives of $\sin(2x)$.) Prove your guess via mathematical induction on n.

4) (15 pts) A number of electronic coins have recently gained value. In particular, a bytecoin is worth 10 cents, a megacoin is worth one dollar, an opticoin is worth one dollar and finally a knightcoin is also worth one dollar. How many different combinations of bytecoins, megacoins, opticoins and knightcoins are worth exactly 20 dollars? Please leave your answer in powers, combinations, factorials, etc. and carefully explain what each expression in your answer and work represent.

5) (10 pts) What is the probability that a randomly chosen positive divisor of 15^{79} is an integer multiple of 45^{30}? Please express your answer as a fraction in lowest terms.

6) (10 pts) Let $f(x) = \frac{3x-2}{x+5}$, with a domain of all reals except x = -5. Determine $f^{-1}(x)$ as well as the domain and range of $f^{-1}(x)$.

7) (12 pts) Define a relation R over the set of rational numbers as follows: R = { (p, q) | p - q has a denominator less than 200 when reduced to lowest terms }. Determine, with proof, whether or not R is (a) reflexive, (b) irreflexive, (c) symmetric, (d) anti-symmetric and (e) transitive.