LA Session – Probability 1 - Solutions

1) 30 tickets are sold in a raffle where 4 prizes will be given. Terri buys 3 of the tickets. What is the probability that Terri wins 0 prizes? 1 prize? 2 prizes? 3 prizes?

Solution

The sample space is $\binom{30}{4}$, because the prizes correspond to any selection of 4 tickets out of the 30 tickets in the raffle.

The number of ways that Terri can choose k winning tickets and 3 – k non-winning tickets is $\binom{4}{k}\binom{26}{3-k}$. Thus, the answer to each of the four questions can be ascertained by plugging in k = 0, 1, 2 and 3:

$$p(W=0)=\frac{\binom{4}{0}\binom{26}{3}}{\binom{30}{4}}, p(W=1)=\frac{\binom{4}{1}\binom{26}{2}}{\binom{30}{4}}, p(W=2)=\frac{\binom{4}{2}\binom{26}{1}}{\binom{30}{4}}, p(W=0)=\frac{\binom{4}{3}\binom{26}{0}}{\binom{30}{4}}$$

Plugging into a calculator, we can get decimal approximations for each and we see:

P(W=0) ~ 0.09487319832147419
P(W=1) ~ 0.04743659916073709
P(W=2) ~ 0.00569239189928845
P(W=3) ~ 0.00014595876664842

2) The integers from 1 to 10, inclusive, are partitioned at random into two sets of five elements each. What is the probability that 1 and 2 are in the same set?

Solution

Without loss of generality, let 1 be in set A. 2 can be located in 9 other slots, 4 in set A and 5 in set B. The probability that 2 is in set A is $\frac{4}{9}$, since 2 is equally likely to be in any of the 9 slots and 4 of these belong to set A.

Another solution is to view the sample space as the number of ways to choose 4 items out of 9, since these are all of the possible ways to fill up set A. Of these $\binom{9}{4}$ possible ways to complete set A, there are $\binom{8}{3}$ of these that include 2 because once 2 is selected, there are only 3 free slots to fill out of 8 possible options. It follows that the desired probability is $\frac{\binom{8}{3}}{\binom{9}{4}}=\frac{8\times7\times6\times4\times3\times2}{9\times8\times7\times6\times3\times2}=\frac{4}{9}$.

3) Sam's probability of getting an A on an individual test is 80%. If he takes ten tests, what is the probability he gets As on exactly 7 of those tests?

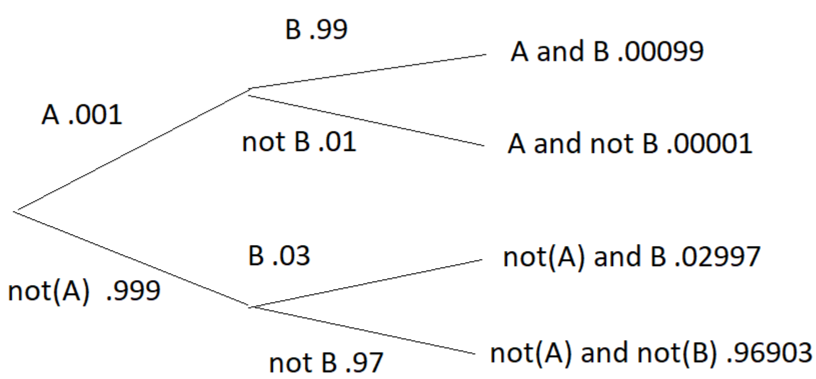
Solution

This is a binomial probability distribution with n = 10, k = 7 and p = .8. Thus, the desired probability is $\binom{10}{7}(.8)^7(.2)^3$~0.2013265920000001, so roughly 20%.

4) Suppose that one person in 1,000 people has a rare genetic disease. There is an excellent test for the disease; 99% of the people with the disease test positive and only 3% of the people who don't have it test positive. What is the probability that someone who tests positive has the disease? What is the probability that someone who tests negative does not have the disease?

Solution

Let A be the event that a person has the disease and let B be the event that they test positive for the disease. Using the given information, we have the following probability tree:



We aim to find p(A | B). Note that we must find p(B) first:

$$p(B) = p(A \cap B) + p(\bar{A} \cap B) = .00099 + .02997 = .03996$$

Now, we can answer the question:

$$p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{.00099}{.03996} \sim 0.02477477477477$$

5) Suppose E and F are events in a sample space and p(E) = 2/3, p(F) = 3/4, and p(F | E) = 9/10. Find p(E | F).

Solution

$p(F|E) = \frac{p(F \cap E)}{p(E)}$, thus plugging in the given information we have $\frac{9}{10} = \frac{p(F \cap E)}{\frac{2}{3}}$. It follows that $p(F \cap E) = \frac{3}{5}$. Now, let us find p(E | F):

$$p(E\,|F) = \frac{p(E \cap F)}{p(F)} = \frac{\frac{3}{5}}{\frac{3}{4}} = \mathbf{\frac{4}{5}}$$