

LA Session – Counting 2

- 1) Consider an ant that is walking on a Cartesian grid, starting at $(0,0)$ and ending at $(15, 18)$. The ant always chooses to walk exactly one unit either up or to the right (towards his destination) whenever he arrives at a Lattice point. (A Lattice point is a point with integer coordinates.) Thus, from $(0,0)$ he either walks to $(1, 0)$ or $(0, 1)$. If the ant is not allowed to go to the points $(6, 8)$ and $(11, 15)$, how many different paths can he take on his walk?
- 2) A class contains 18 girls and 14 boys. For all parts of this question, each boy and girl are distinguishable from one another. Answer the following questions:
- a) In how many ways can a committee of one boy and one girl be chosen?
 - b) In how many ways can a committee of five students be chosen?
 - c) In how many ways can a committee of four girls and three boys be chosen?
 - d) In how many ways can a committee of six students be chosen such that all the students on the committee are the same sex?
 - e) In how many ways can the girls and boys form a line where no two boys are standing next to one another?
 - f) How many committees of seven students contain at least two girls?
- 3) How many solutions does the equation $a + b + c + d + e + f = 30$ have if each variable must be a non-negative integer and $a \leq 3$, $b \leq 7$ and $d \geq 8$?
- 4) How many solutions does the equation $a + b + c + d + e + f + g + h \leq 40$ have if each variable must be a non-negative integer?
- 5) A class has $2n$ students who must be split up into pairs. We consider two sets of pairs S and T different if at least one pair in the set S isn't a pair in the set T . A pair is unordered, so we consider the pair $(1, 2)$ and $(2, 1)$ to be the same pair. How many different sets of pairs can the class be split up into, in terms of n ? (For example, with $n = 2$, the answer is 3. Sets are $\{(1, 2), (3, 4)\}$, $\{(1, 3), (2, 4)\}$ and $\{(1, 4), (2, 3)\}$.)
- 6) How many integers in between 1 and 10^7 are divisible by 15, 35, or 77?