LA Session - Exam 2 Review (Fall 2018 Exam 2 - Edited)

1) (8 pts) What is the result of the following matrix computation?

$$\begin{bmatrix} 9 & 6 & 3 \\ 7 & 4 & 7 \end{bmatrix} \times \begin{bmatrix} 10 & 2 \\ 5 & 7 \\ 10 & 3 \end{bmatrix} =$$

2) (10 pts) What is a closed-form expression in terms of the positive integer *n* for the summation below:

$$\sum_{i=n}^{2n} \left(\sum_{j=1}^{2i} ij \right)$$

3) (14 pts) Prove using induction on n that for all positive integers n, $11 \mid (13^n - 2^n)$.

4) (15 pts) Find all integer solutions to the equation $177x + 78y = 18$.

5) (15 pts) Find the ***sum of the divisors*** of 225,000, leaving your answer in ***prime factorized form.*** (Hint: First prime factorize the given integer. Use the formula from class to express the sum of divisors as a product of some fractions. It may be helpful for you to use the following factoring formula: $x^3 - 1 = (x - 1)(x^2 + x + 1)$. Then, cancel as necessary and express what remains in prime factorized form. This will require a bit of hand calculation, but nothing that you calculate by hand should exceed 1,000. It also may help you to know that $5^4 = 625$.)

6) (10 pts) Let t_n be defined as follows: $t_0 = -1$, $t_1 = 2$, $t_n = 5t_{n-1} - 6t_{n-2}$, for all integers $n \geq 2$. Prove, using strong induction on n, that for all non-negative integers n, $t_n = 4(3^n) - 5(2^n)$.

7) (10 pts) Let a be a positive real number with $a \geq 2$. Using induction on n, prove for all non-negative integers n that

$$\sum_{i=0}^{n} a^i < a^{n+1}$$

8) (8 pts) Let $a_1, a_2, a_3, \ldots$ form an arithmetic sequence with $a_{10} = 13$ and $a_{30} = 53$. Determine the sum of the first 20 terms of the sequence, $\sum_{i=1}^{20} a_i$. Put a box around your final answer.

9) (8 pts) Prove for all positive integers a, b and c: if $a = \gcd(b, c)$, then $a^2 \mid (bc)$.

10) (2 pts) Which office supply company bought the naming rights to the Staples Center in LA?