<u>LA Session - Solving Linear Integer Equations Solutions</u>

1) Find all integer solutions to the equation 847x + 539y = 7.

This equation has no integer solutions because 847x + 539y = 77(11x + 7y). Thus, the left-hand side is divisible by 77 but the right-hand side is not. It follows there are no integer solutions for x and y to this equation.

2) (a) Find all integer solutions to the equation 108x + 47y = 1.

108 = 2 x 47 + 14
47 = 3 x 14 + 5
14 = 2 x 5 + 4
5 = 1 x 4 + 1

5 – 1 x 4 = 1
5 – (14 – 2 x 5) = 1
3 x 5 – 1 x 14 = 1
3(47 – 3 x 14) – 1 x 14 = 1
3 x 47 – 9 x 14 – 1 x 14 = 1
3 x 47 – 10 x 14 = 1
3 x 47 – 10(108 – 2 x 47) = 1
3 x 47 – 10 x 108 + 20 x 47 = 1
23 x 47 – 10 x 108 = 1

It follows that one solution is (-10, 23). Since 108 and 47 are relatively prime, we can write all integer solutions as follows:

$\{ (x, y) \mid x = -10 + 47c, y = 23 - 108c, c \in Z \}$

(b) Find all integer solutions to the equation 108x + 47y = 9.

Take the last line of the Extended Euclidean from the previous question:

23 x 47 – 10 x 108 = 1

Now, multiply through by 9:

(9 x 23) x 47 – (10 x 9) x 108 = 1 x 9
207 x 47 – 90 x 108 = 9

It follows that all solutions can be expressed as:
$\{ (x, y) \mid x = -90 + 47c, y = 207 - 108c, c \in Z \}$

If you want to "re-center" the solution, here's a form of the same set that looks "nicer":
$\{ (x, y) \mid x = 4 + 47c, y = -9 - 108c, c \in Z \}$

(c) Find 47^{-1} mod 108. (Note: Answer must be in between 0 and 107, inclusive.)

Once again, take the last line from the Extended Euclidean in part (a):

23 x 47 – 10 x 108 = 1

Take this equation mod 108 to get:

23 x 47 ≡ 1 (mod 108)

It follows that $47^{-1} \equiv 23 \pmod{108}$