

**LA Session - Exam 1 Review (Fall 2018 Exam 1 - Edited)**

1) (8 pts) Complete filling out the truth table below that evaluates the logical expression. For ease of reading, **please use 0 for false and 1 for true.**

$$(p \vee \bar{q}) \rightarrow (\bar{r} \rightarrow \bar{q}).$$

$p$	$q$	$r$	$p \vee \bar{q}$	$\bar{r} \rightarrow \bar{q}$	$(p \vee \bar{q}) \rightarrow (\bar{r} \rightarrow \bar{q})$
0	0	0			
0	0	1			
0	1	0			
0	1	1			
1	0	0			
1	0	1			
1	1	0			
1	1	1			

2) (8 pts) Using the laws of logic, show that two following expressions are logically equivalent.

$$(p \wedge (p \vee \bar{r} \vee q)) \vee ((q \wedge r) \vee (q \wedge \bar{r}))$$

$$\bar{p} \rightarrow q$$

Note: You may not use all of the rows shown below.

Step	Reason
1. $(p \wedge (p \vee \bar{r} \vee q)) \vee ((q \wedge r) \vee (q \wedge \bar{r}))$	Given
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	

3) (8 pts) Use the rules of inference to make the following argument:

$$s \rightarrow (p \rightarrow r)$$

$$s \vee t$$

$$\bar{t}$$

$$p \vee q$$

-----

$$q \vee r$$

Step	Reason
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	

4) (5 pts) For an open statement  $P(x, y)$ , it is known that  $\exists x[\forall y(P(x, y))]$ . Is it necessarily true that  $\forall y[\exists x(P(x, y))]$ ? If it is the case, prove it, if the assertion is false, create a specific open statement  $P(x, y)$  for which  $\exists x[\forall y(P(x, y))]$  is true and  $\forall y[\exists x(P(x, y))]$  is false and explain why the first is true and the second is false.

5) (10 pts) Prove or disprove the following assertion about finite sets  $A$  and  $B$ , taken from the positive integers (Note:  $\wp$  indicates Power Set.):

$$\wp(A) - \wp(B) \subseteq \wp(A - B).$$

6) (10 pts) Prove or disprove the following assertion about finite sets  $A$ ,  $B$ , and  $C$  taken from the positive integers:

$$(A - C) - (B - C) \subseteq (A - B)$$

7) (10 pts) Consider finite sets A, B and C where we know the cardinalities of the following sets:

$$|A \cap C| = 0$$

$$|A \cup B| = 20$$

$$|B \cap C| = 5$$

$$|A \cup B \cup C| = 22$$

Determine  $|C|$ . (Note: Solutions that only use a Venn-Diagram to determine the answer will receive a maximum of 5/10 points. Only solutions that use a formal proof involving the Inclusion-Exclusion principle will earn full credit.)

8) (3 pts) Let  $S = \{2x^2 + x - 6 = 0 | x \in Z\}$ . (Note: Recall that Z is the set of integers.) Explicitly list each element that belongs to the set S. Put a circle around your final answer.

9) (1 pt) At what time in the afternoon does one develop a five o'clock shadow? \_\_\_\_\_