

LA Session - Logic

1) Fill out the following truth table:

| p | q | r | $(\neg q \vee r)$ | $\neg(\neg q \vee r)$ | $(p \wedge \neg(\neg q \vee r))$ | $p \wedge q$ | $(p \wedge \neg(\neg q \vee r)) \vee (p \wedge q)$ |
|-----|-----|-----|-------------------|-----------------------|----------------------------------|--------------|--|
| F | F | F | | | | | |
| F | F | T | | | | | |
| F | T | F | | | | | |
| F | T | T | | | | | |
| T | F | F | | | | | |
| T | F | T | | | | | |
| T | T | F | | | | | |
| T | T | T | | | | | |

2) Use the laws of logic to prove the two following expressions are logically equivalent:

(a) $p \wedge (\overline{\overline{r} \wedge \overline{p}} \wedge (\overline{p} \rightarrow s))$

(b) p

3) Use the laws of implication to complete the following argument:

1. $p \vee q$
 2. $p \rightarrow s$
 3. $q \rightarrow r$
 4. \overline{r}
 5. $s \rightarrow (t \wedge u)$
-

t

(Note: Numbers 1 – 5 are the premises and proposition below is what is to be deduced from those premises.)

4) Prove or disprove the following statements. Use the domain of real numbers for each variable, unless otherwise stated in the problem.

- a) $\exists x[x^2 + 4x + 3 > 2x^2 + 5x + 6]$
- b) $\forall x \in Z^+[(x + 1)^2 > x^2]$
- c) $\exists x[\forall y(y^2 - 3xy + x^2 \geq 0)]$
- d) $\exists x[\forall y(y^2 - 3xy + x^2 > 0)]$
- e) $\exists x \in Z^+[\exists y \in Z^+(x^y \text{ has precisely 7 divisors})]$