Sample Relation Questions

1) If $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, determine the number of relations on A that are (a) reflexive; (b) symmetric; (c) reflexive and symmetric; (d) reflexive and contain $(1, 2)$; (e) symmetric and contain $(1, 2)$; (f) anti-symmetric; (g) anti-symmetric and contain $(1, 2)$; (h) symmetric and anti-symmetric; (i) reflexive, symmetric and anti-symmetric.

2) Consider the following relation R defined over the set of positive integers:

$$R = \{(x,y) \mid x/y = 4 \vee y/x = 4\}$$

Determine if the relation R is (i)reflexive, (ii)irreflexive, (iii)symmetric, (iv)anti-symmetric, and (v)transitive.

3) If relations R and S defined over the set $A \times A$ are antisymmetric, is $(R \cup S)$ necessarily antisymmetric?

4) Define a relation $T \subseteq N \times N$ such that $T = \{(a,b) | a \in A \wedge b \in A \wedge a - b = 2c+1$ for some integer c}. (N is the set of non-negative integers.)

a) Prove that this relation is not reflexive.
b) Prove that this relation is symmetric.
c) Define the term anti-transitive as the following:

Given a set A and a relation R,
if for all $a,b,c \in A$, $(aRb \wedge bRc \wedge cRa) \Rightarrow (a = b \vee b = c)$

Prove that the relation T is anti-transitive.